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# THE DETERMINATION OF CRITICAL ROUGHNESS HEIGHT FOR BOUNDARY LAYER TRANSITION

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# THE DETERMINATION OF CRITICAL ROUGHNESS HEIGHT FOR BOUNDARY LAYER TRANSITION

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ABSTRACT: A method is presented which allows a determination of the critical roughness height required to produce premature boundary layer transition. This method is applicable to cases involving compressibility, heat transfer, and pressure gradient. The results are presented in graphic form which allows a relatively simple application of the method.

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Captain, USN
Commander

A. E. Seigel
A. E. SEIGEL
By direction

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## SYMBOLS

| | |
|---|---|
| $C_F$ | mean skin friction coefficient |
| $k$ | height of the roughness element |
| $M$ | Mach number |
| $n$ | correlation number defined by equation (11) |
| $p$ | static pressure |
| $R$ | gas constant in perfect gas law |
| $Re_\theta$ | Reynolds number based on boundary layer momentum thickness and flow conditions which exist just outside of the boundary layer |
| $Re_k$ | Reynolds number based on roughness height and flow conditions which exist just outside of the boundary layer |
| $R_{k_k}$ | critical roughness Reynolds number - based on roughness height and flow condition which exist within the boundary layer at a distance k from the wall |
| $S$ | enthalpy function |
| $T$ | absolute temperature |
| $u$ | velocity component tangential to the wall |
| $x$ | coordinate tangential to the wall |
| $y$ | coordinate normal to the wall |
| $\beta$ | pressure gradient parameter |
| $\gamma$ | ratio of specific heats, $\gamma = 1.4$ |
| $\eta$ | similarity variable from reference (9) |
| $\theta$ | boundary layer momentum thickness |
| $\mu$ | coefficient of viscosity |
| $\nu$ | kinematic viscosity |

SYMBOLS (Cont'd.)

| | |
|---|---|
| $\rho$ | mass density |
| $\sigma$ | Mach number function |

**Subscripts**

| | |
|---|---|
| e | local flow conditions just outside the boundary layer |
| k | local flow conditions within the boundary layer at a normal distance k from the wall |
| o | free stream stagnation value |
| w | conditions at the wall |

INTRODUCTION

It is often desirable when conducting aerodynamic experiments to insure that the boundary layer flow over the body is turbulent, or to insure that transition from laminar to turbulent flow occurs at a specified point on the body. One method of inducing premature boundary layer transition on a model is by applying roughness to the surface of the body. In this report a method is presented which allows the determination of the minimum height of surface roughness required to produce premature boundary layer transition, provided a certain parameter is assumed as transition criterion. The method may be applied to compressible laminar boundary layer with pressure gradients and heat transfer at the surface.

In the present method it is assumed that a criterion for roughness affected boundary layer transition can be formed by defining a critical roughness Reynolds number, $R_{k_k} = \frac{\rho_k u_k k}{\mu_k}$, where k is the height of the roughness element and subscript k indicates the flow properties in the boundary layer a distance k from the surface. Thus, if the parameter $R_{k_k}$ is smaller than some experimentally determined critical value, no effect will be made on the boundary layer transition location. Von Doenhoff and Horton (reference (1)) and Smith and Clutter (reference (2)) have satisfactorily correlated subsonic flow transition measurements by utilizing the concept of a critical roughness Reynolds number as defined in this report. In an attempt to correlate results from supersonic boundary layer transition experiments, several different parameters have been proposed by various investigators, references (3), (4), (5), (6), (7), and (8). In the reports by Braslow, Knox, and Horton (reference (7)) and Jackson and Czarnecki (reference (8)), low supersonic boundary layer transition results were correlated by using the critical roughness Reynolds number $R_{k_k}$. This correlation appeared quite satisfactory. Because of simplicity and lack of more reliable supersonic boundary layer transition results, the criterion for roughness induced boundary layer transition has been assumed to be $R_{k_k}$, the critical roughness Reynolds number.

In order to know the flow properties within the boundary layer which are necessary to form the critical roughness Reynolds number, the velocity and enthalpy profiles for a compressible laminar boundary layer presented by Cohen and Reshotko in reference (9) have been utilized in the development of the method. The roughness height causing premature transition is then presented in graphical form as $Re_k$, a Reynolds number based on roughness height and flow properties which exist just outside of the boundary layer at the location at which transition is to be initiated, versus $Re_\theta$, the boundary layer momentum thickness Reynolds number.

## DEVELOPMENT OF METHOD

Based upon the assumption that the criterion for premature boundary layer transition is the existence of a critical roughness Reynolds number $R_{k_k}$, the following method for the determination of the critical roughness height can be developed. The critical roughness Reynolds number is defined as:

$$R_{k_k} = \frac{\rho_k u_k k}{\mu_k} \qquad (1)$$

Using the perfect gas law allows the density to be expressed in terms of the pressure and temperature so that equation (1) may be written as:

$$R_{k_k} = \frac{p_k u_k k}{R T_k \mu_k} \qquad (2)$$

Upon writing all of the quantities on the right-hand side of equation (2) in ratio form with the exception of the gas constant, R, equation (2) becomes:

$$R_{k_k} = R_{e_\theta} \frac{p_k/p_e \; u_k/u_e \; k/\theta}{T_k/T_e \; \mu_k/\mu_e} \qquad (3)$$

where $Re_\theta$ is the momentum thickness Reynolds number based on flow properties just outside of the boundary layer.

The assumption that the pressure across the boundary layer is constant makes the pressure ratio, $p_k/p_e$ in equation (3), unity. Also, by the use of Sutherland's formula, the viscosity ratio can be expressed in terms of the temperatures $T_k$ and $T_e$ as:

$$\frac{\mu_k}{\mu_e}=\left(\frac{T_k}{T_e}\right)^{3/2}\frac{1+198.6/T_e}{T_k/T_e+198.6/T_e} \tag{4}$$

The substitution of equation (4) into equation (3) along with the assumption that $p_k/p_e=1$, yields

$$R_{k_k}=R_{e_\theta}\frac{u_k/u_e\; k/\theta\left(T_k/T_e+198.6/T_e\right)}{\left(T_k/T_e\right)^{3/2}\left(1+198.6/T_e\right)} \tag{5}$$

If the temperature profile expression given in reference (9) is utilized, the temperature ratio can be expressed in terms of the velocity ratio and an enthalpy ratio as:

$$\frac{T_k}{T_e}=\left(1+\frac{\gamma-1}{2}M_e^2\right)\left[1+S_k-\sigma\left(\frac{u_k}{u_e}\right)^2\right] \tag{6}$$

where:

$$\sigma=\frac{\frac{\gamma-1}{2}M_e^2}{1+\frac{\gamma-1}{2}M_e^2}$$

Substituting equation (6) into equation (5) and solving for the ratio $Re_\theta/Rk_k$ gives:

$$\frac{R_{e_\theta}}{R_{k_k}}=\frac{\left(1+198.6/T_e\right)\left\{\left(1+\frac{\gamma-1}{2}M_e^2\right)\left[1+S_k-\sigma\left(u_k/u_e\right)^2\right]\right\}^{3/2}\;1}{u_k/u_e\left\{\left(1+\frac{\gamma-1}{2}M_e^2\right)\left[1+S_k-\sigma\left(u_k/u_e\right)^2\right]+198.6/T_e\right\}\;k/\theta} \tag{7}$$

To calculate a value for the ratio $Re_\theta/R_{k_k}$ from equation (7), it is necessary to specify values for $u_k$, the velocity, and $S_k$, the enthalpy function, at a point in the boundary layer at a distance k from the wall. This implies that velocity and temperature profiles through the boundary layer must be specified. Cohen and Reshotko (reference (9)) have presented velocity and enthalpy profiles for the laminar compressible boundary layer including the effect of both heat transfer at the wall and a pressure gradient impressed on the boundary layer by the external flow. These velocity and enthalpy profiles are presented as a function of a non-dimensional similarity parameter $\eta$.

Since equation (7) involves the parameter $k/\theta$, and since it is desirable to express the physical distance from the wall k in terms of the non-dimensional similarity parameter $\eta$, it is desirable to determine the expression for $k/\theta$ in terms of $\eta$. An expression for $k/\theta$ can be obtained by utilizing the relations presented in references (9) and (10) for $y$ and $\theta$, and can be written as:

$$\frac{k}{\theta} = \left(1+\frac{\gamma-1}{2}M_e^2\right)\frac{\int_0^{\eta_k}\left[1+S-\sigma\left(u/u_e\right)^2\right]d\eta}{\int_0^{\infty} u/u_e\left(1-u/u_e\right)d\eta} \qquad (8)$$

By the use of equations (7) and (8), a value for the critical roughness Reynolds number based on flow properties evaluated just outside of the boundary layer can be determined. Hence, if the unit Reynolds number of the flow just outside of the boundary layer is known, a value for the critical roughness height required to produce premature boundary layer transition can be determined.

## CALCULATIONS AND PRESENTATION OF RESULTS

The velocity and enthalpy profiles presented in reference (9) allows a value for $k/\theta$ to be calculated for any specified values of $\eta_k$ and Mach number from equation (8). This value of $k/\theta$ can be substituted into equation (7) along with the appropriate values of $u_k/u_e$ and $S_k$ that correspond to that value of $\eta_k$ that has been specified. Further, substitution of values for $M_e$ and $T_e$ allows a value for $Re_\theta/R_{k_k}$ to be calculated.

The velocity and temperature profiles given in reference (9) are identified by a wall enthalpy function, $S_w$, and a pressure gradient parameter $\beta$. The wall enthalpy function is defined as:

$$S_w = \frac{T_w}{T_o} - 1 \qquad (9)$$

A method for obtaining a value for $\beta$ will be discussed under the section entitled "General Application of Method." Velocity and temperature profiles are presented in reference (9) for five values of $S_w$ in the range $-1 \leq S_w \leq 1$. The $S_w = -1$ corresponds to a wall temperature of absolute zero while the $S_w = 1$ corresponds to a wall temperature twice the free stream stagnation temperature. For each of five values of $S_w$, the velocity and enthalpy profiles are presented for a range of values of $\beta$ from 2 to less than -0.1. The $\beta = 2$ represents an infinitely favorable pressure gradient, while $\beta < 0$ represents adverse pressure gradients.

A value for $Re_k/R_{k_k}$ is obtained from the following equation using the calculated value for the ratio $Re_\theta/R_{k_k}$.

$$\frac{Re_k}{R_{k_k}} = \frac{Re_\theta}{R_{k_k}} \cdot \frac{k}{\theta} \qquad (10)$$

This procedure for calculating values of $Re_k/R_{k_k}$ was carried out using a high-speed digital computer. Velocity and enthalpy profiles presented in reference (9) for all five values of $S_w$ were used. For each value of $S_w$, profiles for a large range of values of $\beta$ representing both favorable and adverse pressure gradients were utilized. Values of $\eta_k$ were chosen from 0.1 to the value corresponding to the outer edge of the boundary layer for increments of $\eta_k = 0.1$ These calculations were carried out for Mach numbers from 0 to 5 in increments of 1 Mach number. Because Sutherland's formula was used to represent the viscosity temperature relationship, it can be seen from equation (7) that $Re_\theta/R_{k_k}$ is dependent also upon Te. It will be shown later, however, that $Re_\theta/R_{k_k}$ and hence $Re_k/R_{k_k}$ is insensitive to variations of Te so that all of the calculations to be presented were performed for a constant value of Te = 500°R.

The results of these calculations are presented in graphical form as a family of curves of constant Mach number plotted as $Re_\theta/R_{k_k}$ versus $Re_k/R_{k_k}$. A separate chart is presented for each combination of values of $S_w$ and $\beta$ used in the calculations. These charts are presented as Figures 1. The upper limit on the values of $Re_\theta/R_{k_k}$ presented in these figures was dictated by the choice of $\eta_k = 0.1$ as the minimum value of $\eta_k$ for which the calculations were performed. Since the correlation between the roughness height required to produce premature transition and a critical roughness Reynolds number is valid only for roughness elements within the boundary layer, the calculations were performed only for values of $\eta$ equal to or less than that value which corresponds to the boundary layer thickness. This fact dictates the lower limit on the values of $Re_\theta/R_{k_k}$.

To illustrate the insensitiveness of $Re_k/R_{k_k}$ to variations of Te, calculations of the type previously indicated have been performed letting Te vary from 100°R to 1,000°R. The calculations were performed for $S_w = 0$, and $\beta = 0$. The results of the calculations are presented in Figure 2 as a plot of $Re_k/R_{k_k}$ versus Te with a family of curves of constant values of $Re_\theta/R_{k_k}$. These curves are for a Mach number of 5.

Although curves have been presented for Mach numbers from 0 to 5 sufficient data are not currently available to indicate positively that the correlation between roughness height and critical roughness Reynolds number remains valid at hypersonic Mach numbers.

It is known, however, that as the Mach number increases, the distance between the roughness elements and the point of transition increases where the height of the roughness elements used is equal to the critical height. This was pointed out, for instance, by Van Driest and Blumer (reference (6)). Therefore, at hypersonic Mach numbers, it may become difficult to produce transition in the near vicinity of the roughness elements. Even though premature transition can be produced by the application of roughness elements, it usually occurs far downstream of the location of the roughness.

## GENERAL APPLICATION OF METHOD

To apply the results of the method presented, it is necessary to specify values for $S_w$ and $\beta$, the enthalpy and pressure gradient parameters, that correspond to the point at which the roughness elements are to be located. The value for $S_w$ can be determined from equation (9). In order to determine the value of $\beta$, it is more convenient to relate it to the correlation number $n$ defined in reference (10). The relation between $\beta$ and $n$ is given graphically in Figure 3 as a plot of $\beta$ versus $n$. Five curves, one for each of the five values of $S_w$ are presented. The value of $n$ corresponding to the point of interest can be calculated from the relation given in reference (10) as:

$$n = -\frac{\theta^2}{\nu_w}\left(\frac{T_w}{T_e}\right)^2 \frac{T_0}{T_e} \frac{du_e}{dx} \tag{11}$$

This expression can be written in a more convenient form for the purpose of this report as:

$$n = -R_{e_\theta} \frac{\theta}{M_e} \frac{T_w}{T_e} \frac{\mu_e}{\mu_w} \frac{dM_e}{dx} \tag{12}$$

Values for $S_w$ and $\beta$ are now available which will allow the correct chart of $Re_k/R_{k_k}$ versus $Re_\theta/R_{k_k}$ to be found. The relation between $Re_k/R_{k_k}$ and $Re_\theta/R_{k_k}$ is specified by knowing the local Mach number which identifies the correct curve on the chart.

At this point it is necessary to choose the value for the critical roughness Reynolds number, $R_{k_k}$. A review of the experimental investigations reported in references (4), (5), (6), and (7), indicates that a reasonable value for the critical roughness Reynolds number required to initiate premature transition is approximately 700 if three-dimensional roughness elements are used. To move transition to the location of the roughness, a value slightly larger than 700 should be used for the lower supersonic Mach numbers. As was previously mentioned in the section entitled "Calculations and Presentation of Results,"

it may be difficult at the higher supersonic Mach numbers to shift transition to the roughness location. It is a direct process utilizing the proper chart in Figures 1 to obtain a value for $Re_k$ once an appropriate value for $R_{k_k}$ has been chosen. Finally, a value for the roughness height is obtained by dividing the value of $Re_k$ by the local unit Reynolds number.

The application of this method to a specific case will rarely involve values of $S_w$, $\beta$, and Me which correspond exactly to any of the values of these quantities presented in Figures 1. It will therefore be necessary to interpolate using the information in Figures 1 to obtain a value for the roughness height which applies to the specific case. For the local Mach number, Me, a graphic interpolation can be performed, while for the quantities $S_w$ and $\beta$, numerical interpolation is required. From Figure 4, which is a typical plot of $Re_k/R_{k_k}$ versus $S_w$, it can be seen that a linear interpolation should be sufficiently accurate to determine $Re_k/R_{k_k}$ between any two adjacent values of $S_w$ presented in Figures 1. Figure 4 has been presented for a constant local Mach number of 4 and $Re_\theta/R_{k_k}$ of 1. Figure 5 is the same type of plot as Figure 4 with the abscissa changed from $S_w$ to $\beta$ and the curves being a family of constant values of $S_w$. Again it can be seen from this figure that linear interpolation is sufficient except perhaps in certain regions of adverse pressure gradients. Figure 6 shows the variation of $Re_k/R_{k_k}$ with local Mach number. These curves are presented as a family for constant values of $S_w$. All of the curves in Figure 6 are for $\beta = 0$ (zero pressure gradient). Since curves are presented in Figures 1 for every Mach number between 0 and 5, it can be seen from Figure 6 that a linear interpolation should be sufficiently accurate to determine values for $Re_k/R_{k_k}$ at Mach numbers not specified.

Since the width of the roughness band in the streamwise direction does not appear to be a critical parameter, it is suggested that for convenience a narrow band of roughness be used.

## APPLICATION OF METHOD OF A CONE

Since a sharp-nosed cone is quite often of interest to investigators, it was deemed worthwhile mentioning the application of the present method to this specific body shape. The value of $S_w$ can be computed from equation (9) and since a cone is a zero pressure gradient body the value for $\beta$ will be zero. The remaining information needed to enable the determination of the required roughness height to produce premature transition is a value for the local flow properties on the cone just outside of the boundary layer, and the boundary layer momentum thickness, $\theta$. The local flow properties can be obtained from any one of numerous references such as reference (11). The boundary layer momentum thickness for a cone is given in terms of the momentum thickness on a flat plate obtained at the same local Mach number, local Reynolds numbers, and wall to local temperature ratio by the following simple relations:

$$\theta_{cone} = \frac{1}{\sqrt{3}}\,\theta_{flat\ plate} \qquad (13)$$

This equation can be readily developed by applying Mangler's transformation to flow over a flat plate. The flat plate momentum thickness is given by:

$$\theta_{flat\ plate} = \frac{C_F x}{2} \qquad (14)$$

which is obtained by integrating the momentum equation. In equation (14) $C_F$ is the mean skin friction coefficient, and x is the distance measured along the plate from the leading edge to this point of interest. The information in Figures 1 can be used to determine a value for the necessary roughness height after a value of $\theta$ and hence $Re_\theta$ for the cone has been determined.

## CONCLUSION

A method has been presented for determination of the minimum height of surface roughness that causes premature laminar boundary layer transition. The method is based on the assumption that the height of the roughness necessary to accomplish this can be expressed as a critical roughness Reynolds number. The magnitude of the critical roughness Reynolds number has been determined experimentally by several different investigators and is quite consistent for the lower supersonic Mach numbers. Only a small amount of data is available for higher supersonic Mach numbers, and whether the critical roughness Reynolds number as a transition criterion holds, and if so, what its value is in the higher speed range, can only be determined by further experimentation. Since the charts which are presented in this report are independent of the value of the critical roughness Reynolds number, they may be used for any value of it.

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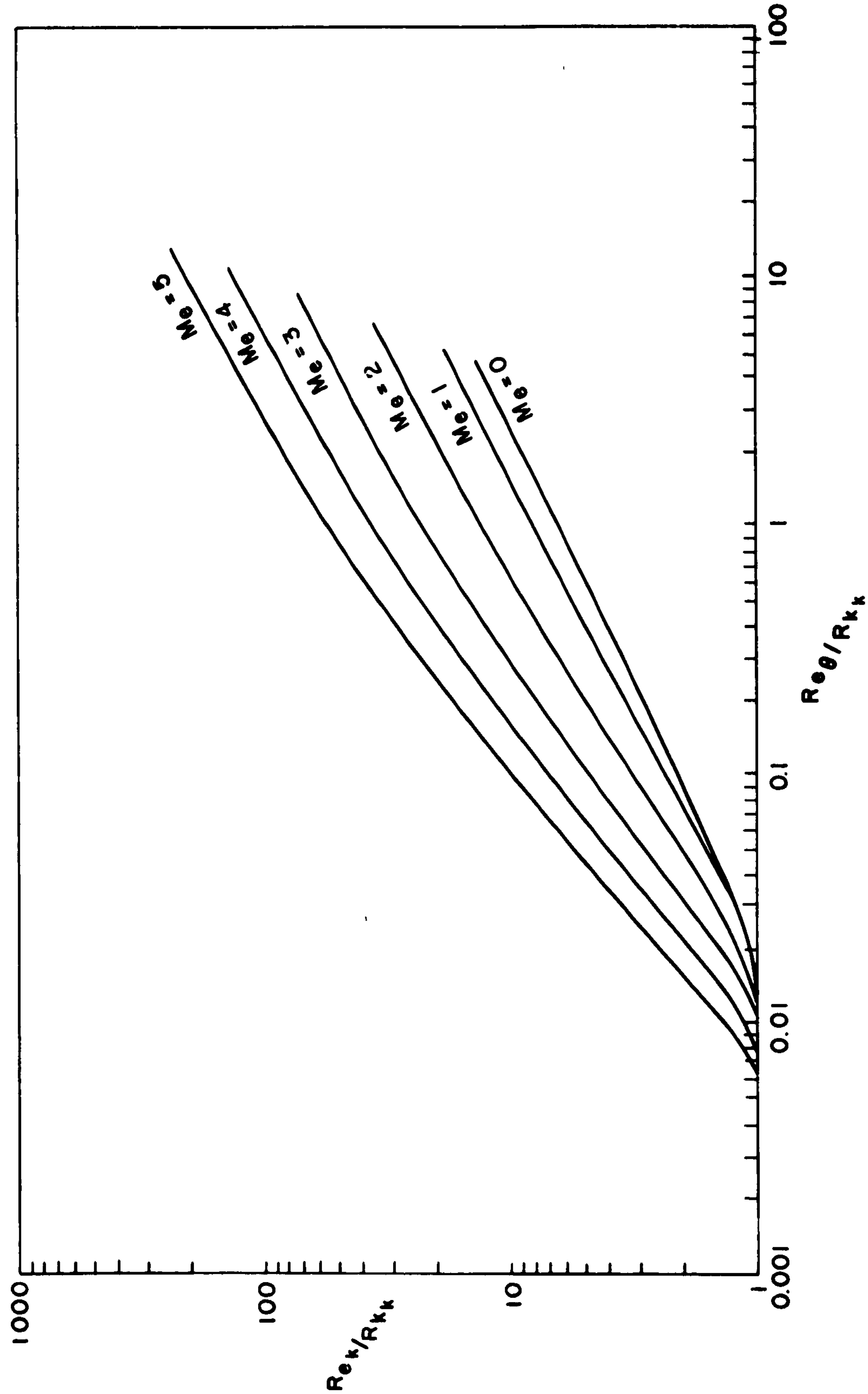
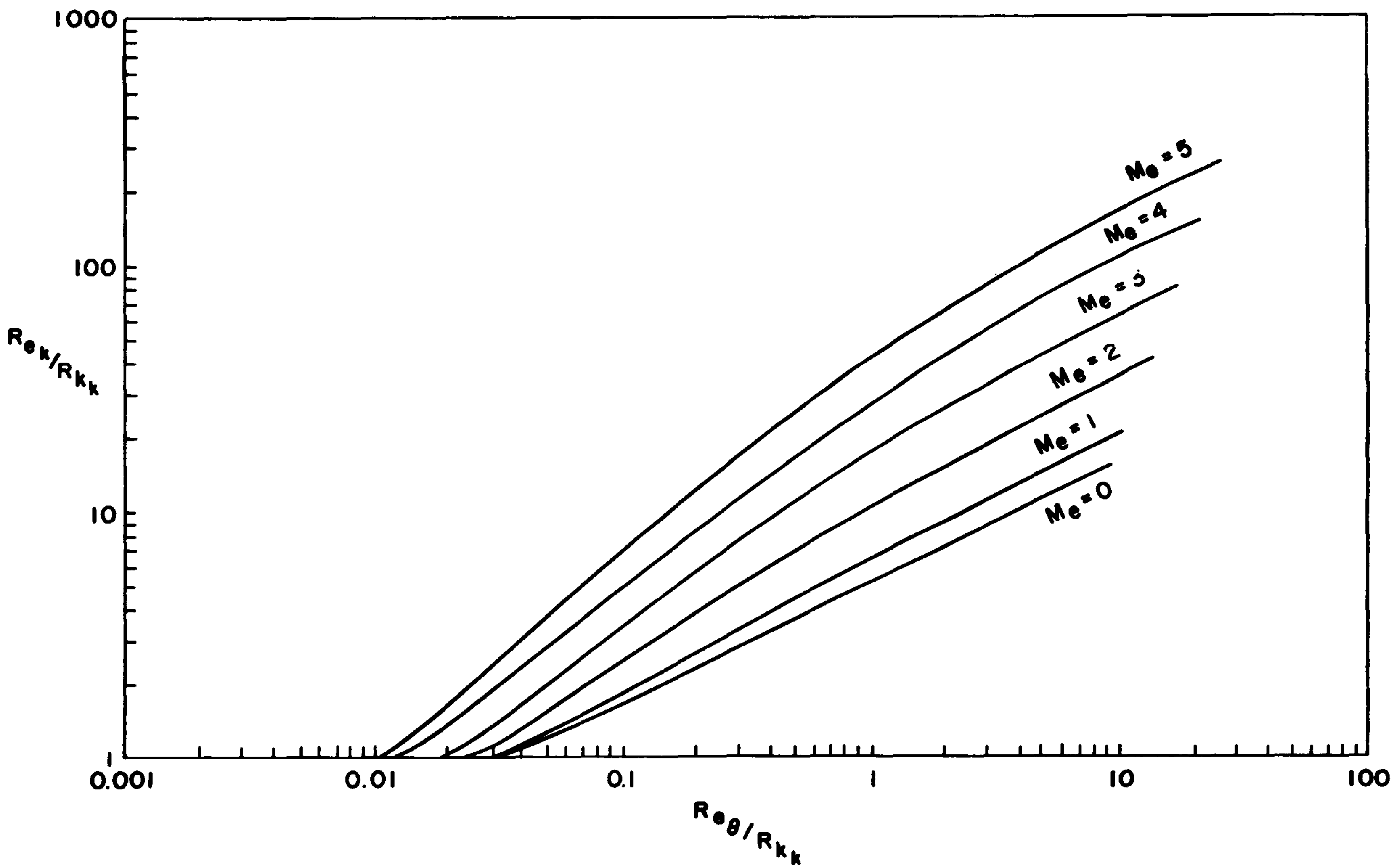


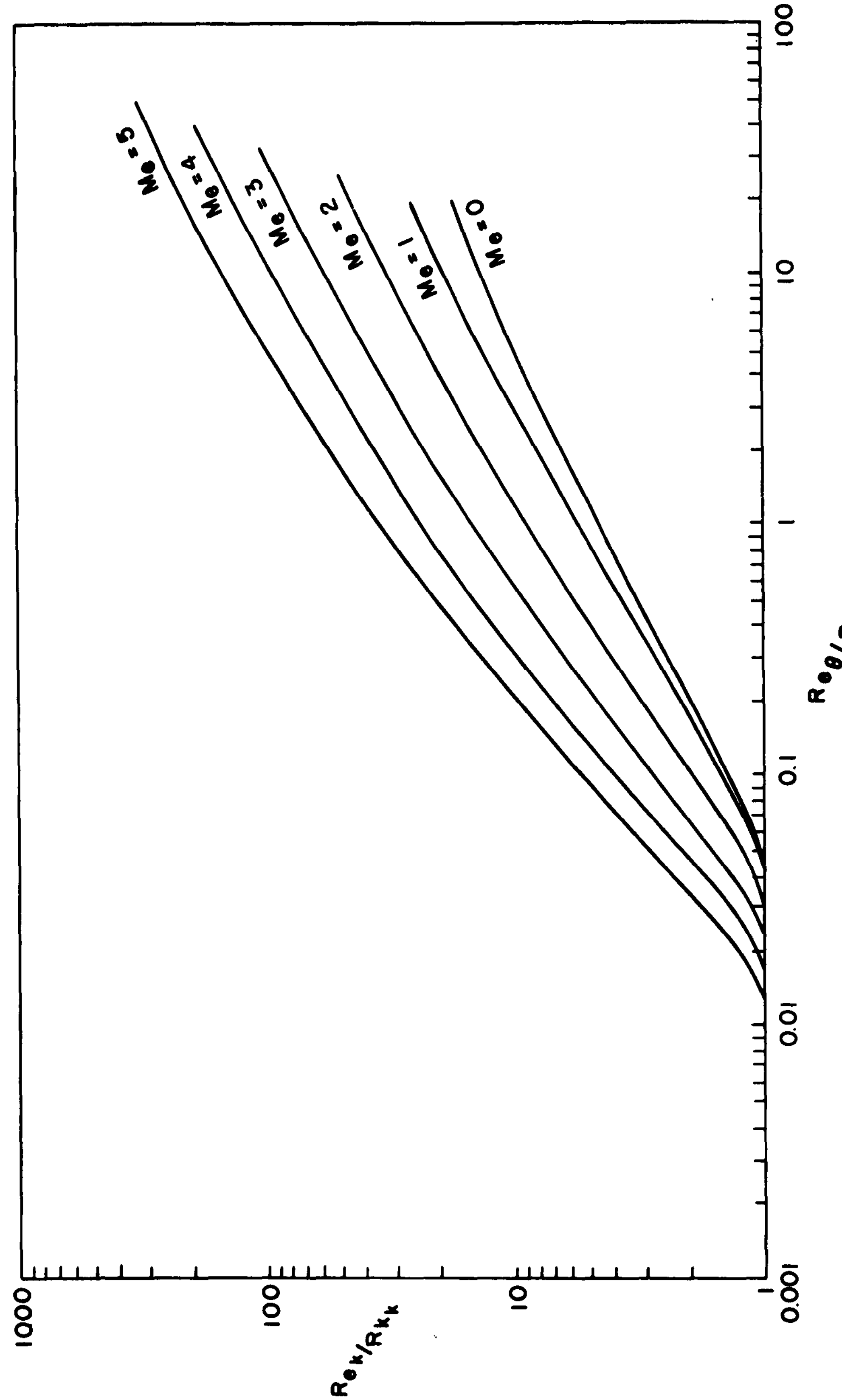

$S_w = 1.0, \beta = 2.0$

FIG. 1 VARIATION OF ROUGHNESS REYNOLDS NUMBER WITH MOMENTUM THICKNESS REYNOLDS NUMBER



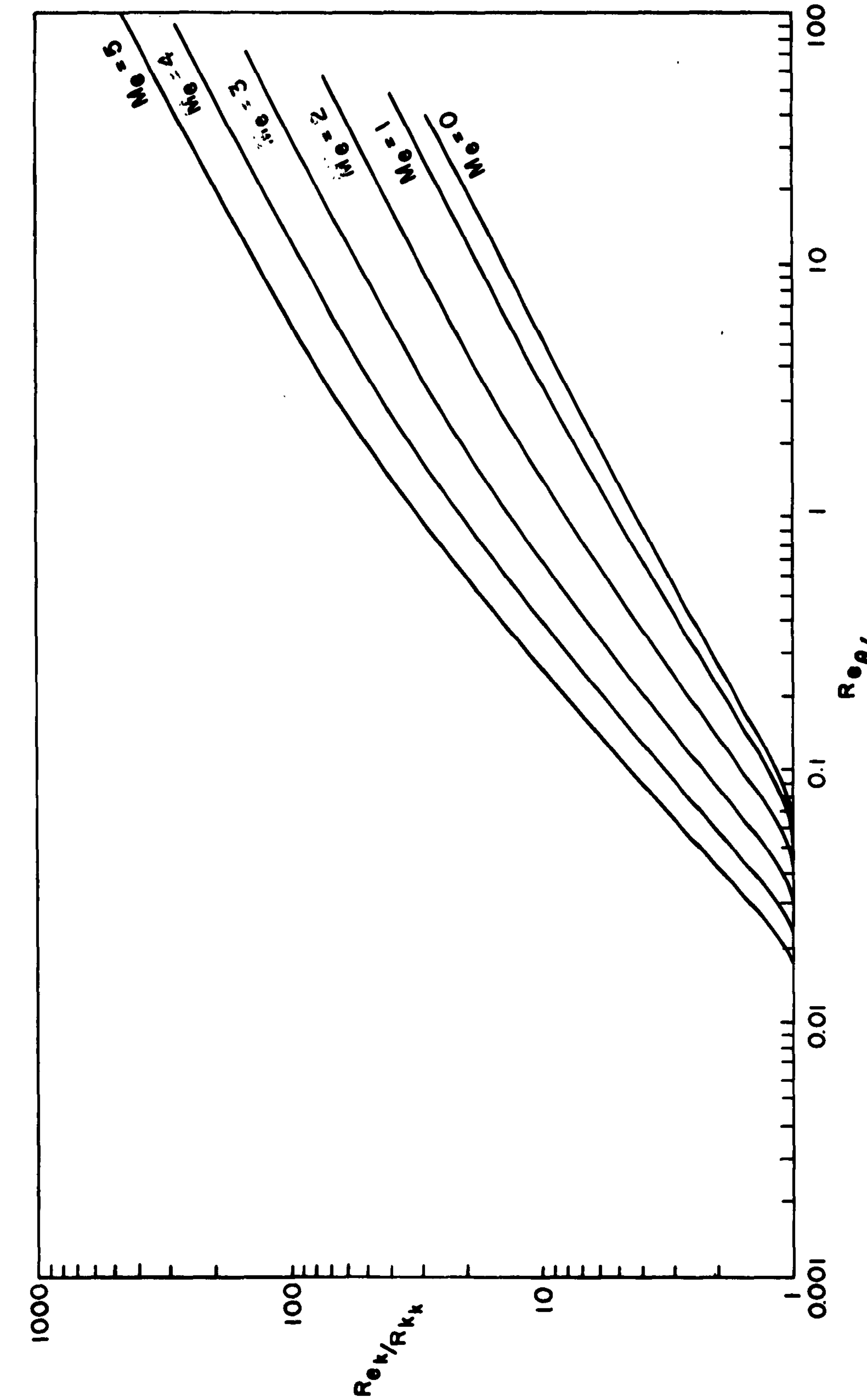

$S_w$=1.0 , $\beta$ = 1.5

FIG. 1 (CONT.) VARIATION OF ROUGHNESS REYNOLDS NUMBER WITH MOMENTUM THICKNESS REYNOLDS NUMBER




$S_w$ = 1.0 , $\beta$ = 1.0

FIG. 1 (CONT.) VARIATION OF ROUGHNESS REYNOLDS NUMBER WITH MOMENTUM THICKNESS REYNOLDS NUMBER




$S_w = 1.0$, $\beta = 0.5$

FIG. 1 (CONT.) VARIATION OF ROUGHNESS REYNOLDS NUMBER WITH MOMENTUM THICKNESS REYNOLDS NUMBER

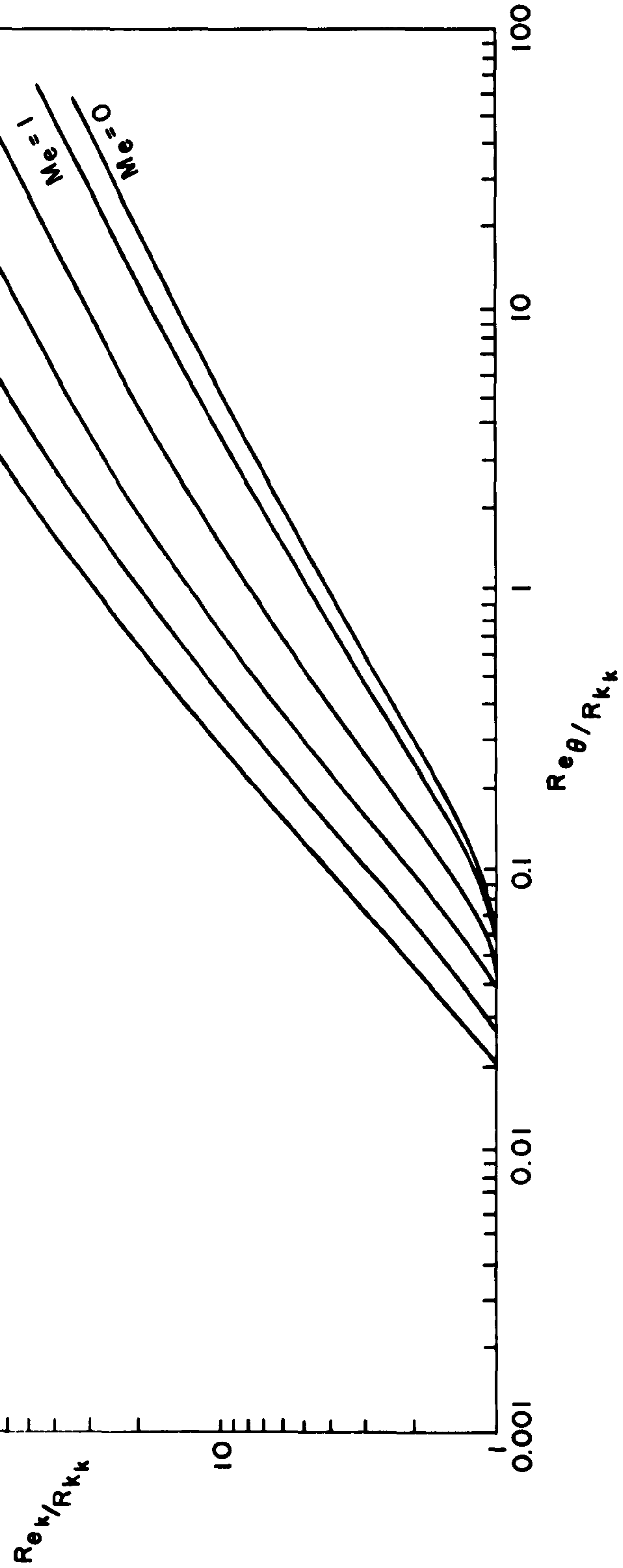



FIG. 1 (CONT.) VARIATION OF ROUGHNESS REYNOLDS NUMBER WITH MOMENTUM THICKNESS REYNOLDS NUMBER

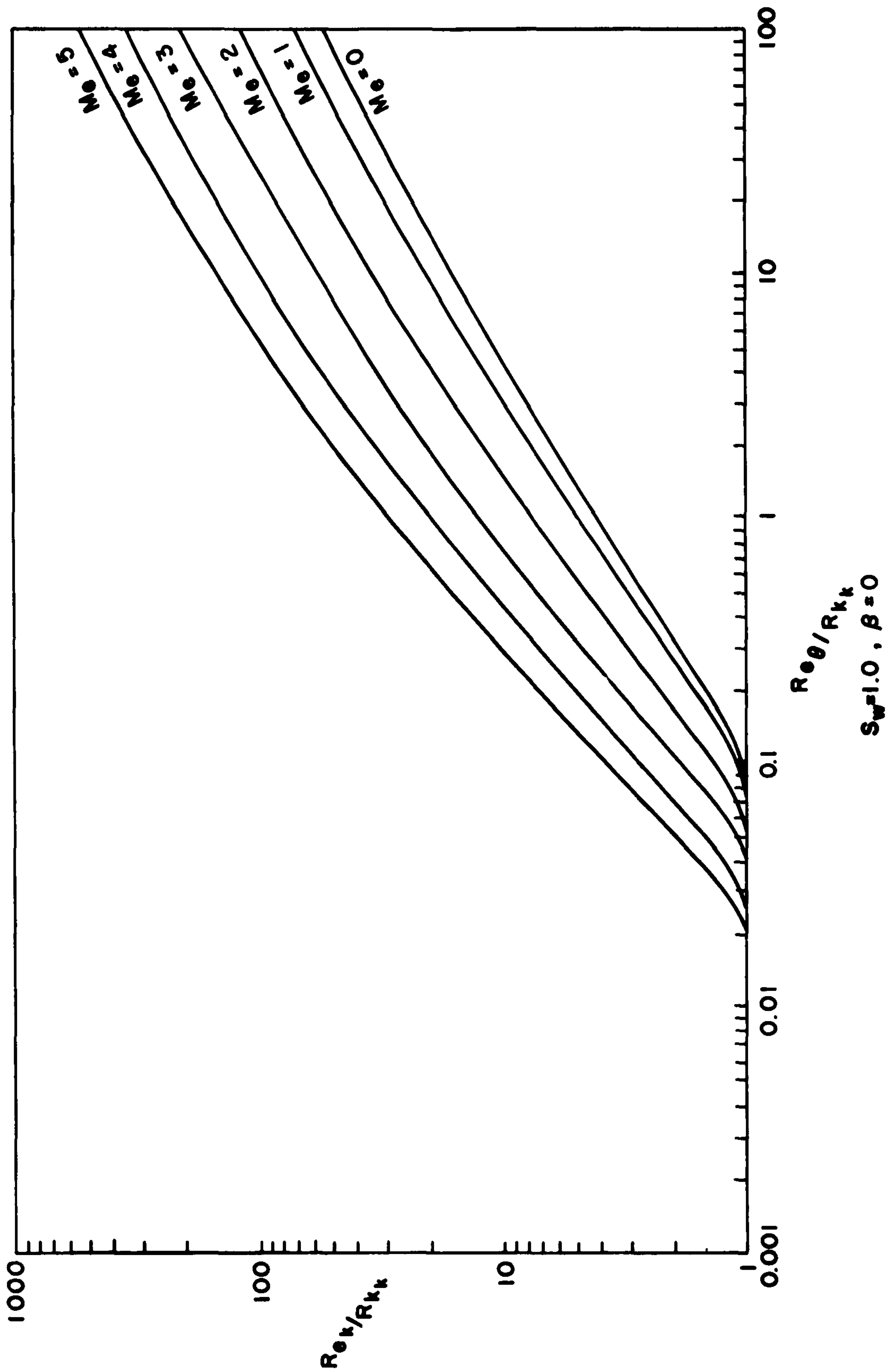



FIG. 1 (CONT.) VARIATION OF ROUGHNESS REYNOLDS NUMBER WITH MOMENTUM THICKNESS REYNOLDS NUMBER

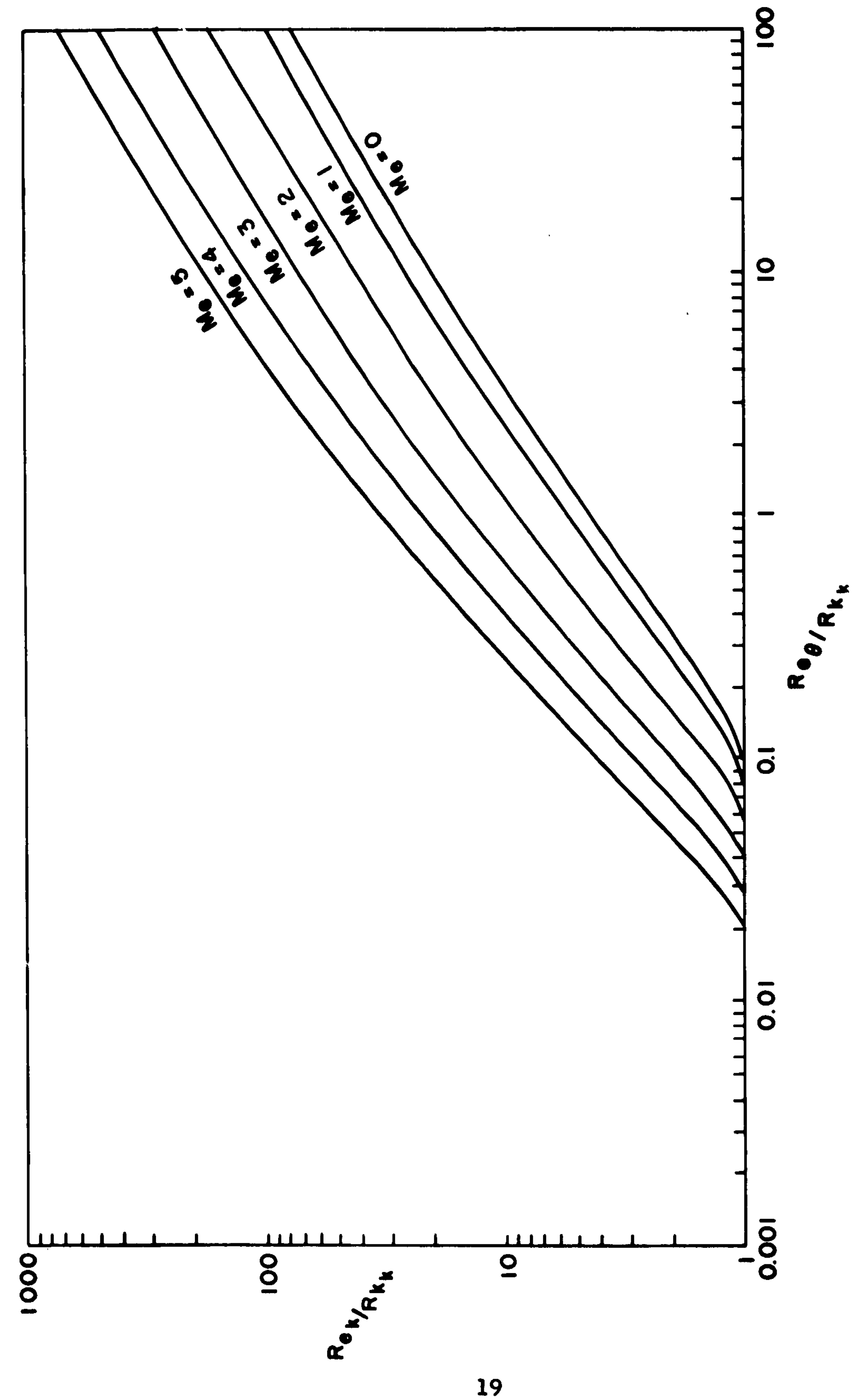
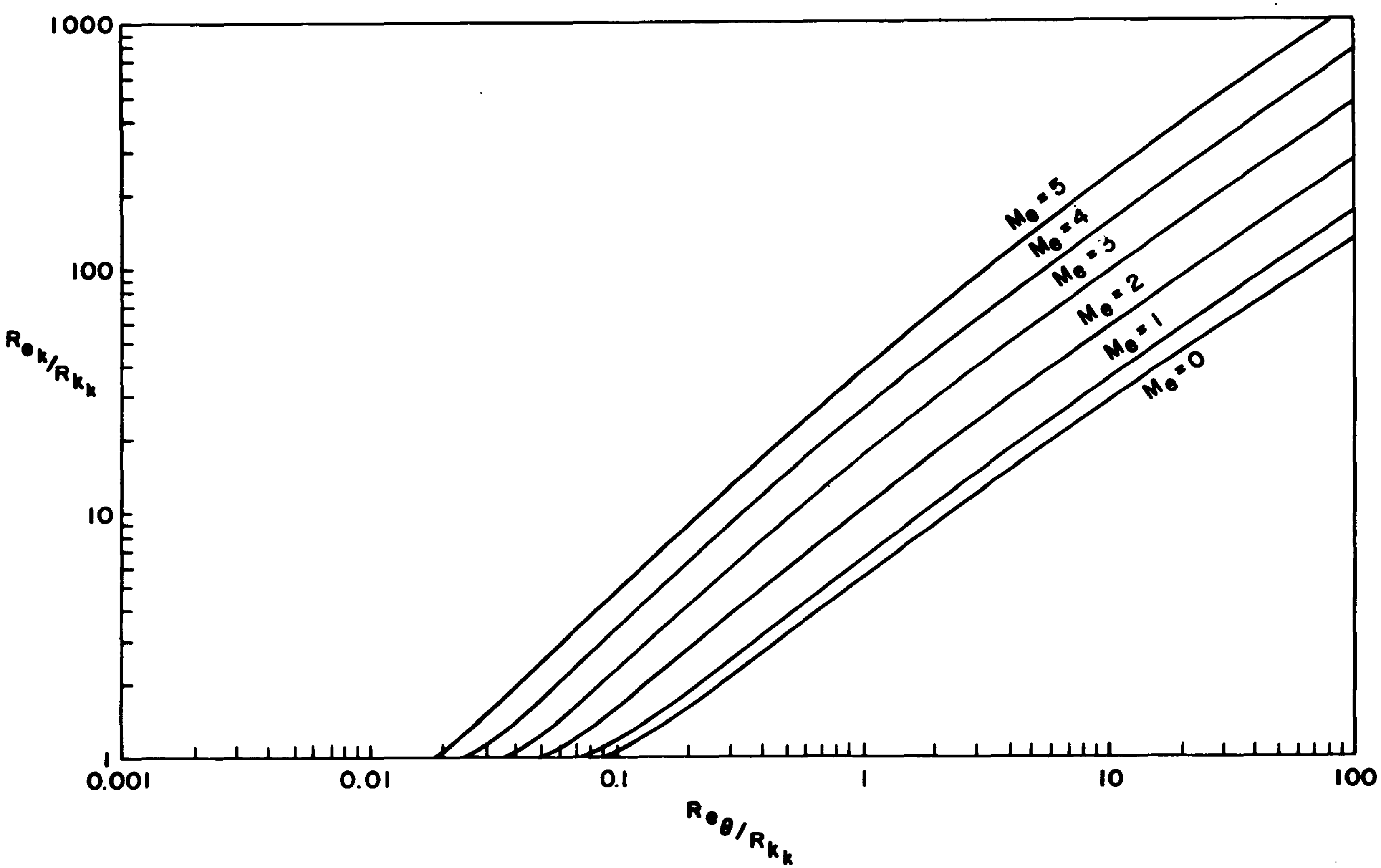
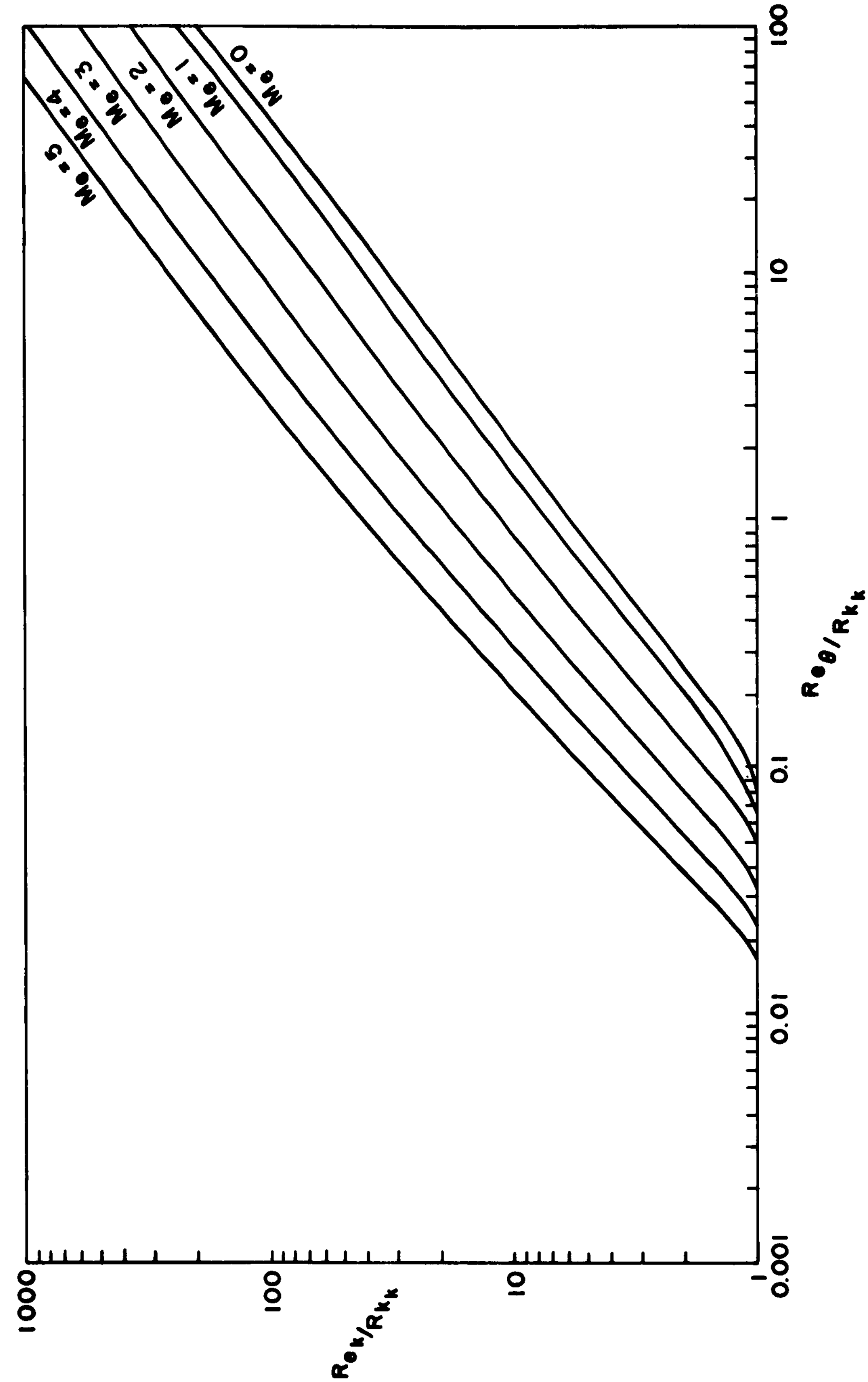



FIG. I (CONT.) VARIATION OF ROUGHNESS REYNOLDS NUMBER WITH MOMENTUM THICKNESS REYNOLDS NUMBER




$S_w=1.0$ , $\beta=-0.1295$

FIG. I (CONT.) VARIATION OF ROUGHNESS REYNOLDS NUMBER WITH MOMENTUM THICKNESS REYNOLDS NUMBER




$S_w = 1.0$ , $\beta = -0.1305$

FIG. 1 (CONT.) VARIATION OF ROUGHNESS REYNOLDS NUMBER WITH MOMENTUM THICKNESS REYNOLDS NUMBER

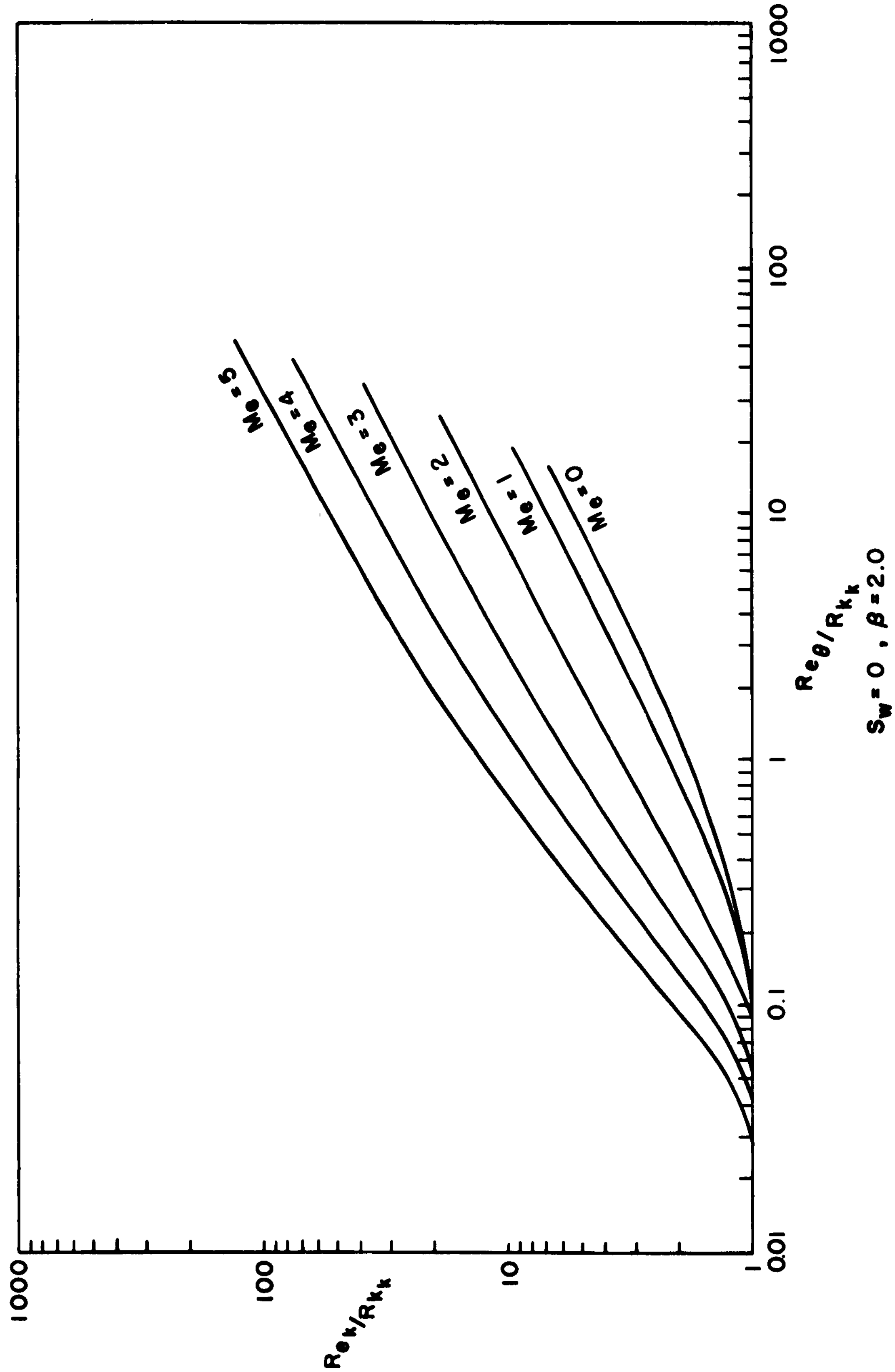



FIG. 1 (CONT.) VARIATION OF ROUGHNESS REYNOLDS NUMBER WITH MOMENTUM THICKNESS REYNOLDS NUMBER

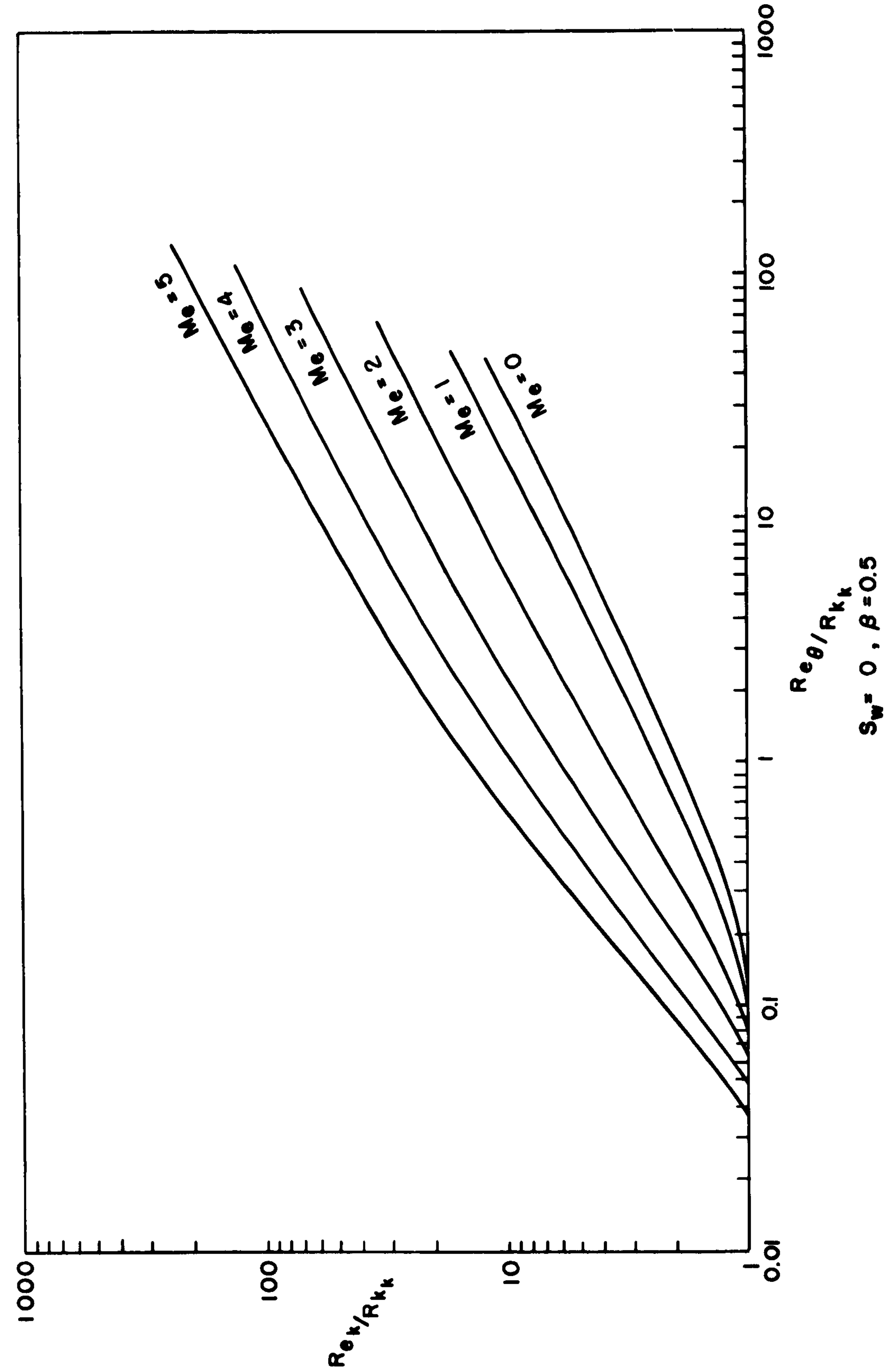
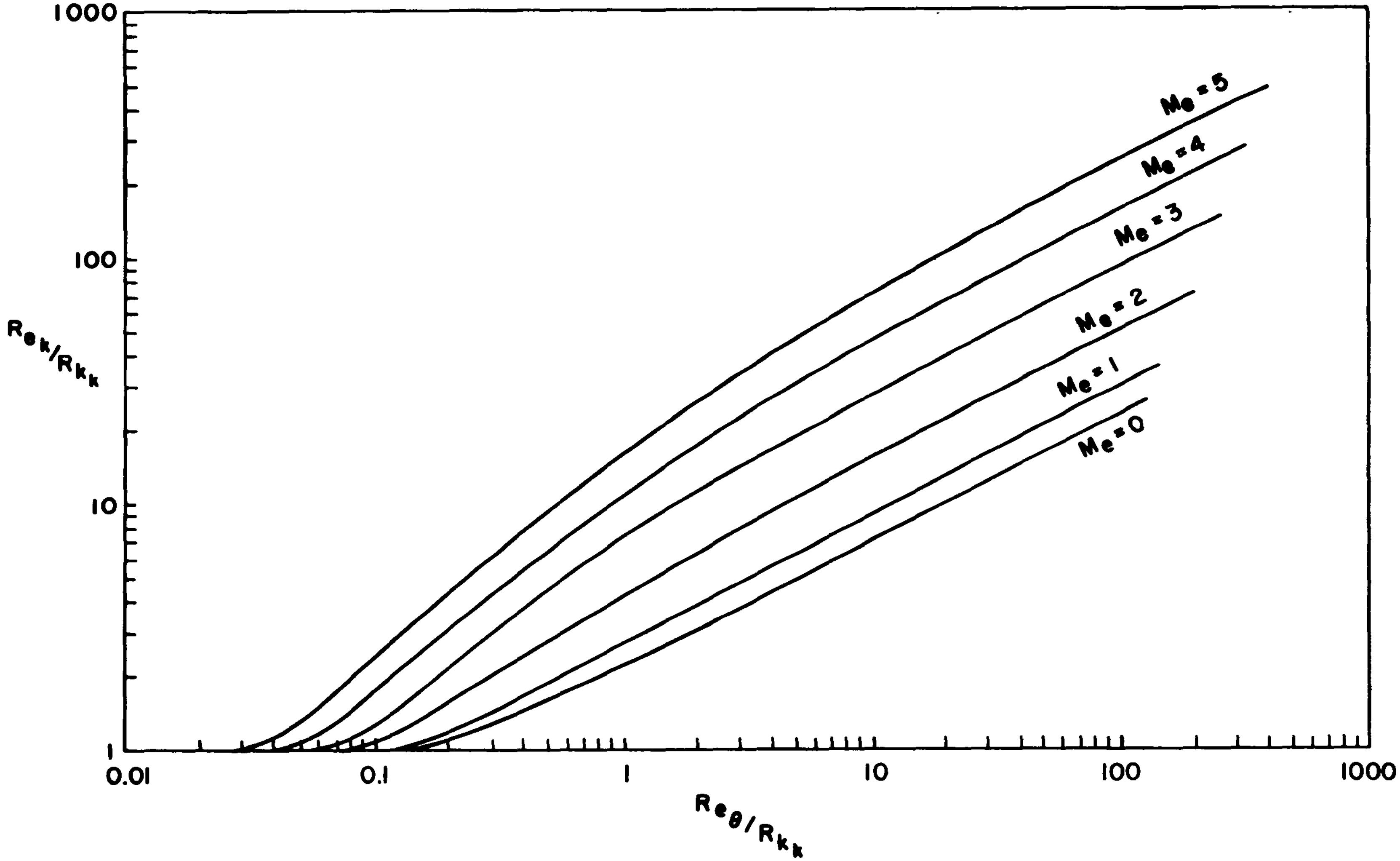
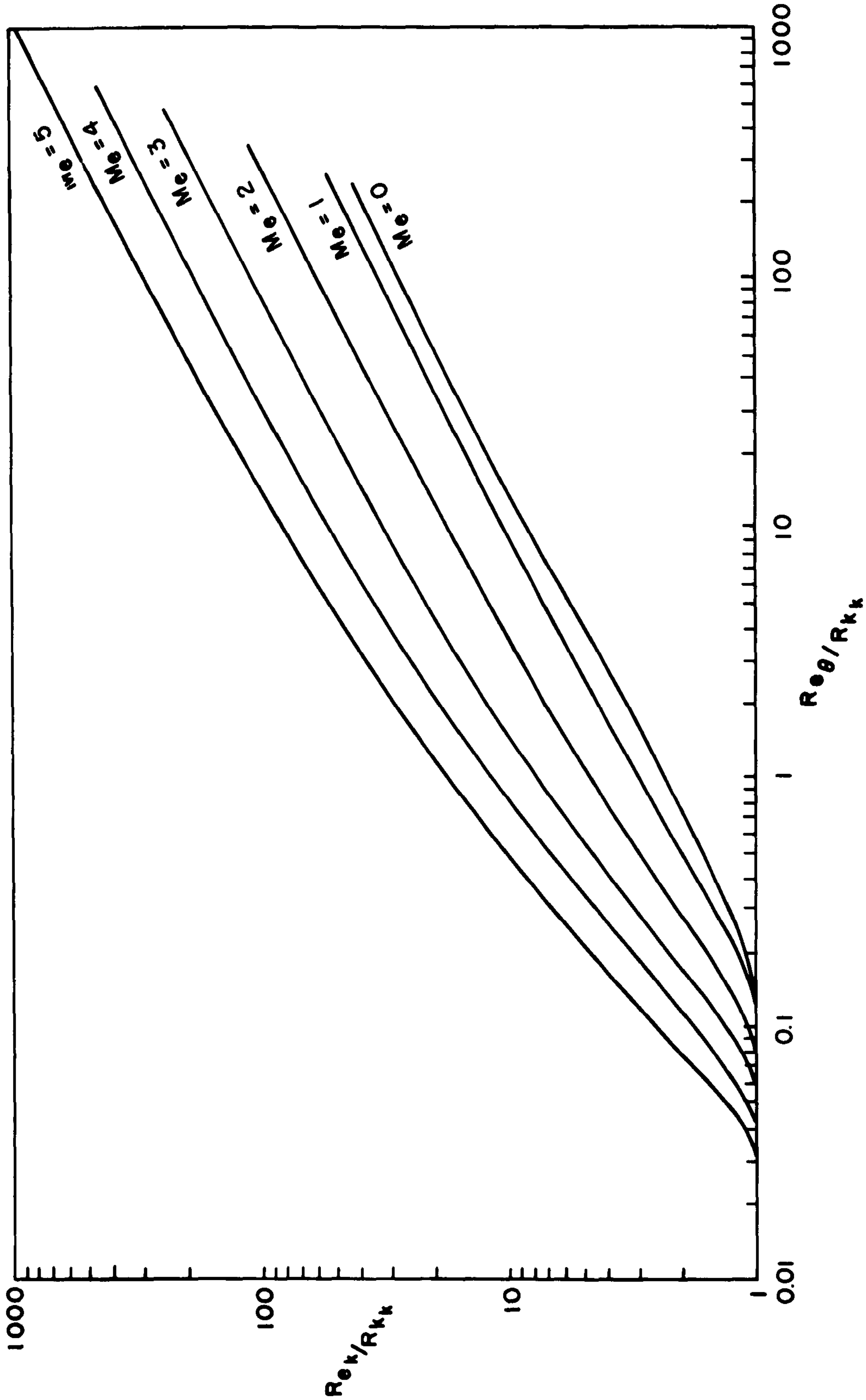



FIG. I (CONT.) VARIATION OF ROUGHNESS REYNOLDS NUMBER WITH MOMENTUM THICKNESS REYNOLDS NUMBER



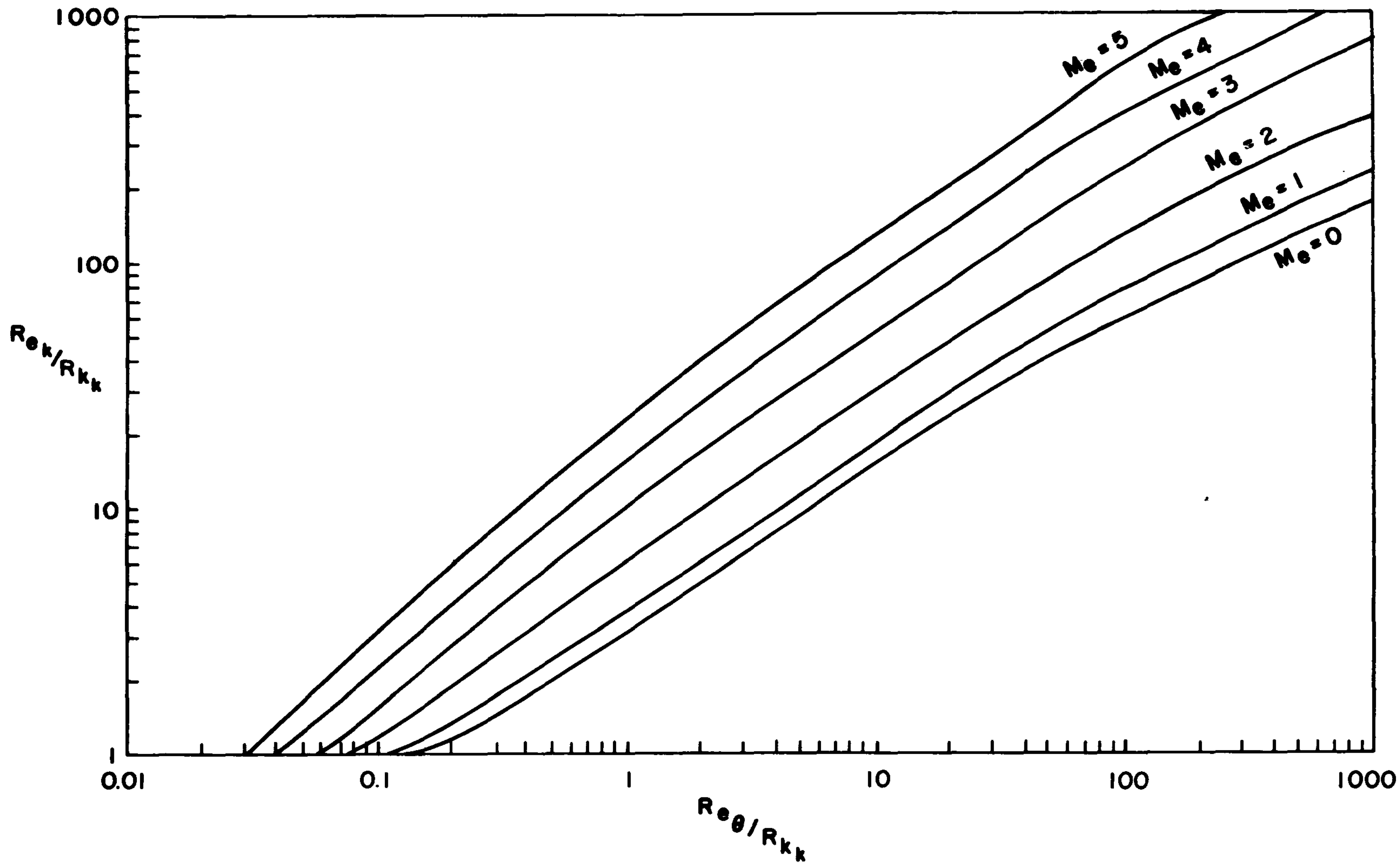

$S_w = 0$ , $\beta = 0$

FIG. 1 (CONT.) VARIATION OF ROUGHNESS REYNOLDS NUMBER WITH MOMENTUM THICKNESS REYNOLDS NUMBER



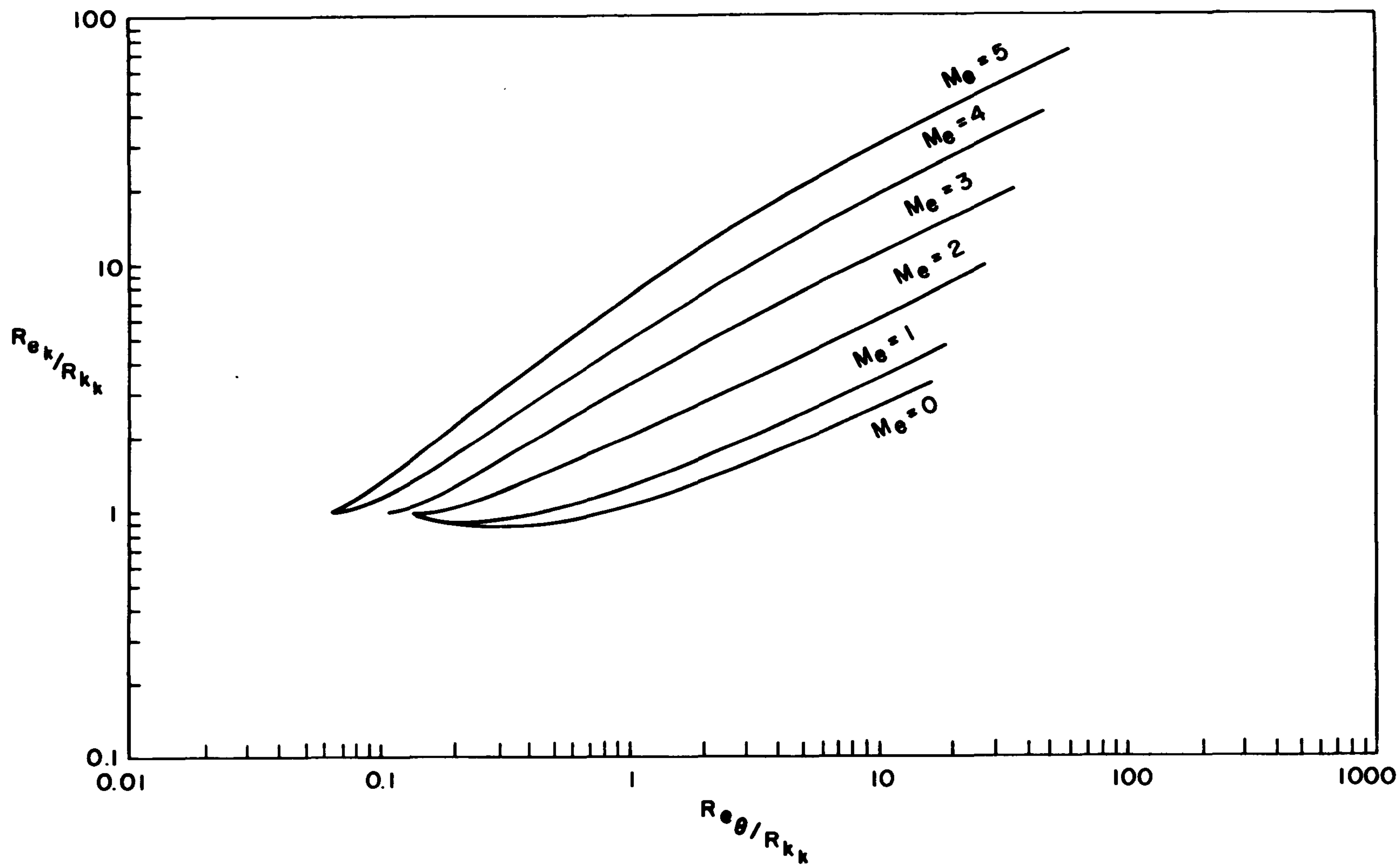

$S_w = 0$ , $\beta = -0.14$

FIG. 1 (CONT.) VARIATION OF ROUGHNESS REYNOLDS NUMBER WITH MOMENTUM THICKNESS REYNOLDS NUMBER




$S_w = 0$ , $\beta = -0.1988$

FIG. 1 (CONT.) VARIATION OF ROUGHNESS REYNOLDS NUMBER WITH MOMENTUM THICKNESS REYNOLDS NUMBER




$S_w = -0.4$, $\beta = 2.0$

FIG. 1 (CONT.) VARIATION OF ROUGHNESS REYNOLDS NUMBER WITH MOMENTUM THICKNESS REYNOLDS NUMBER

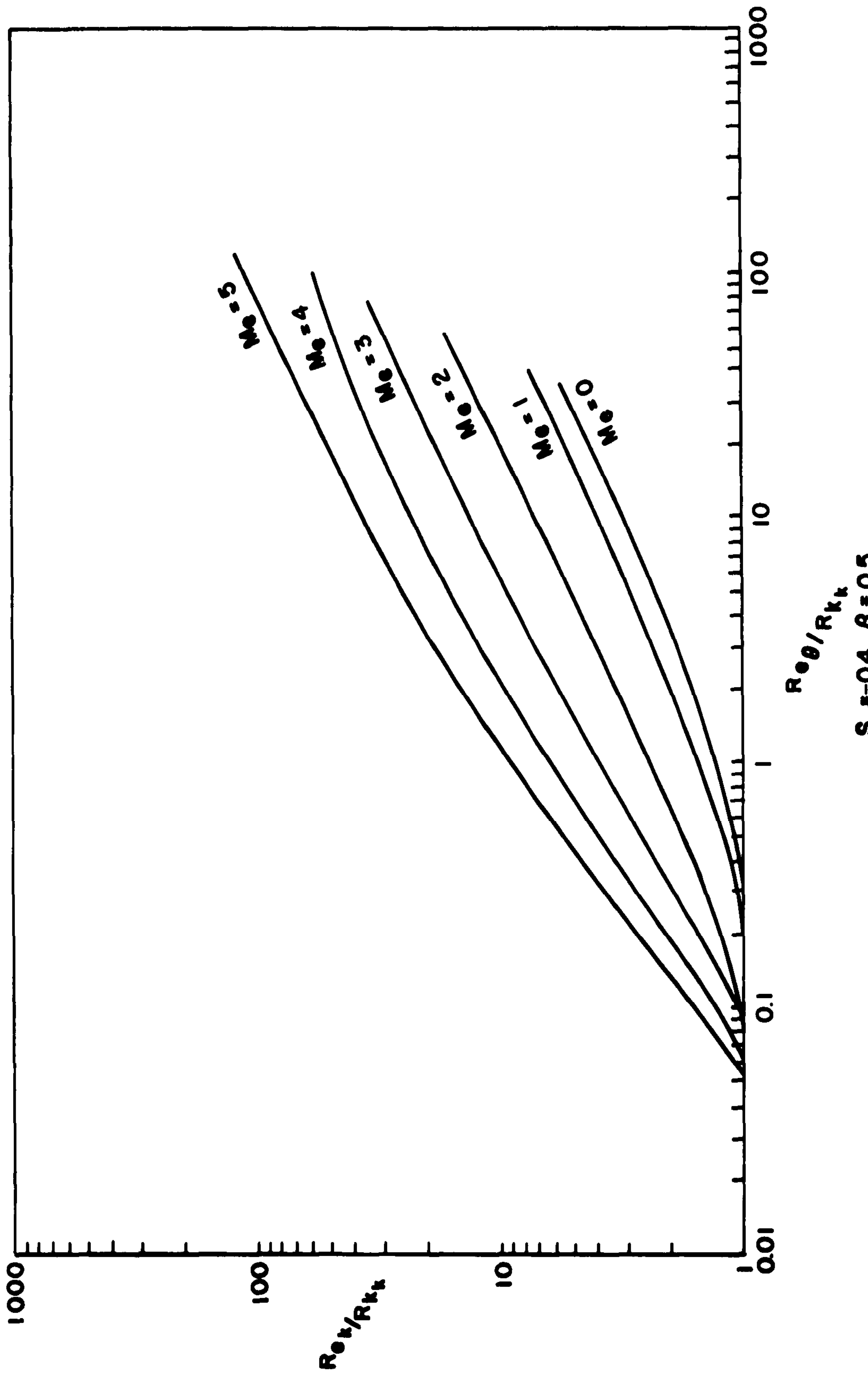
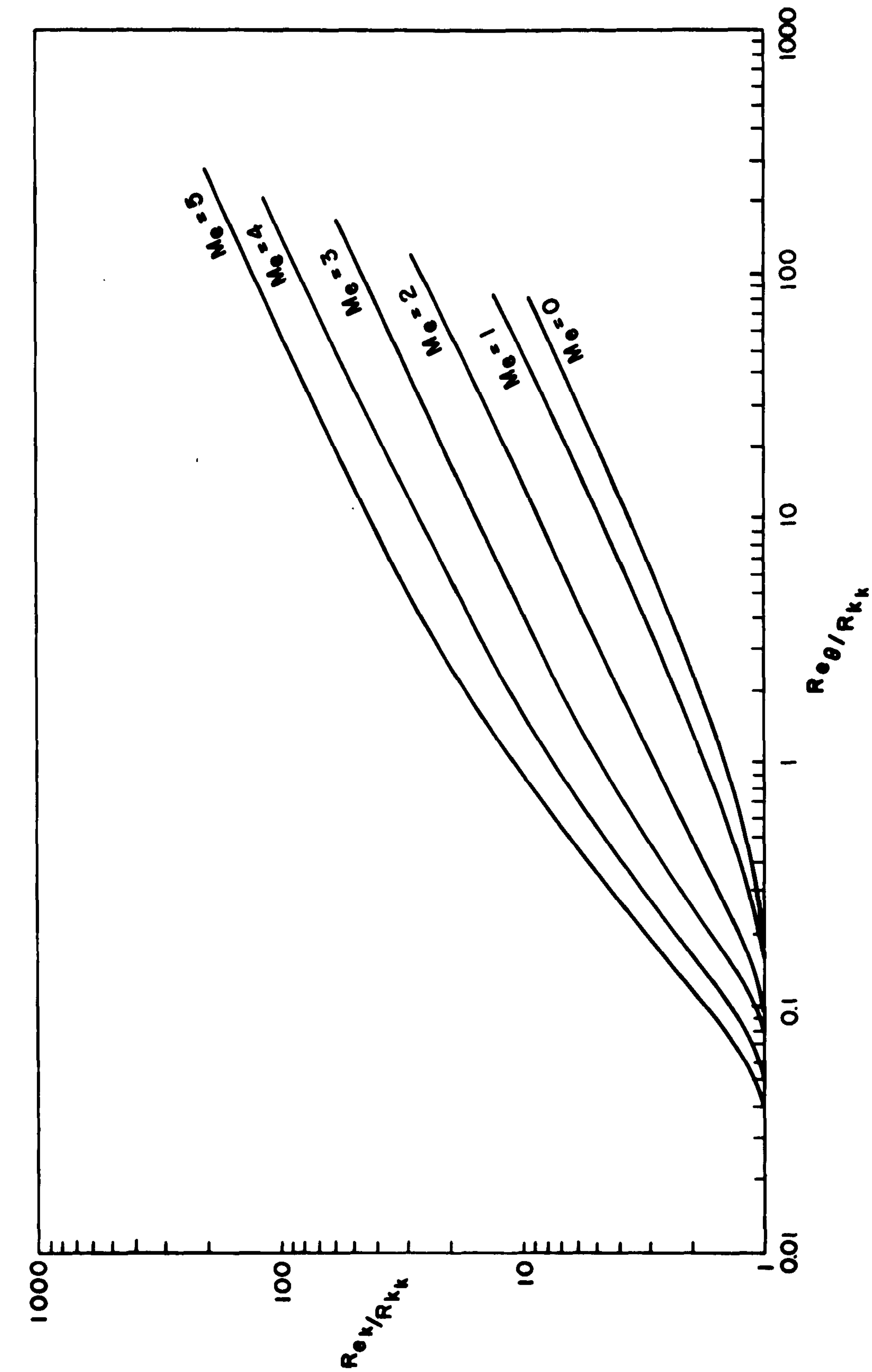
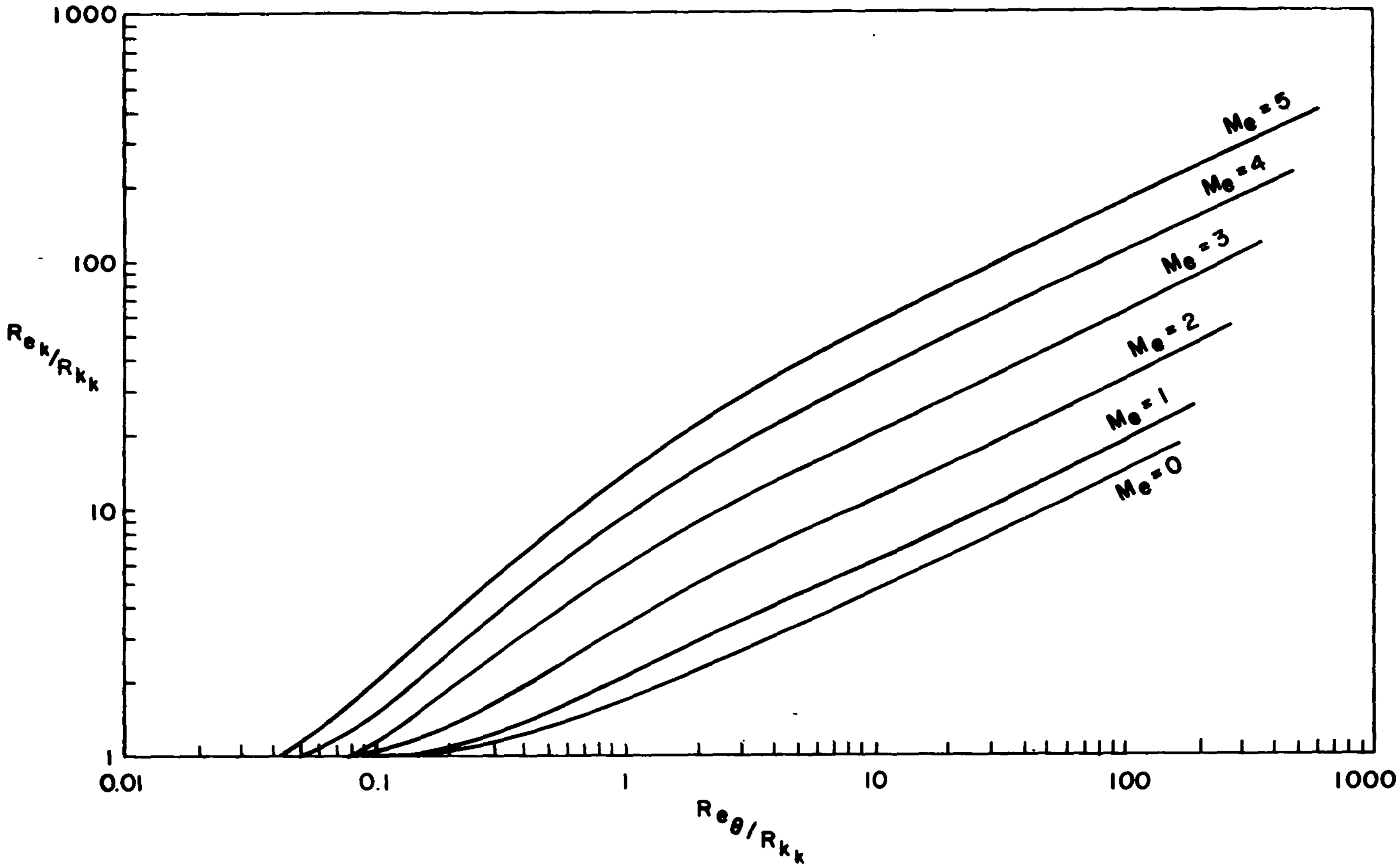



FIG. 1 (CONT.) VARIATION OF ROUGHNESS REYNOLDS NUMBER WITH MOMENTUM THICKNESS REYNOLDS NUMBER



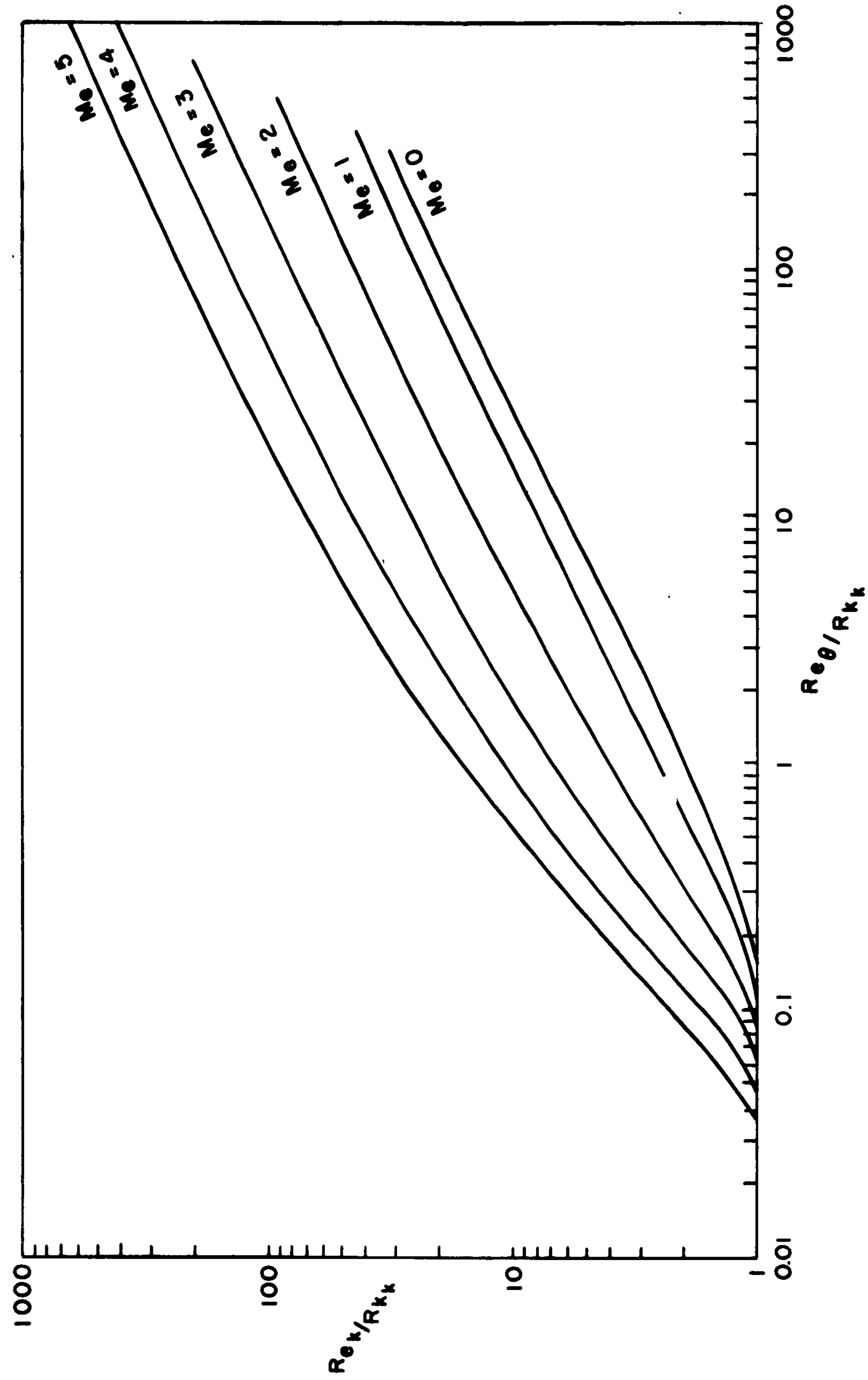

$S_w=-0.4,\ \beta=0$

FIG. 1 (CONT.) VARIATION OF ROUGHNESS REYNOLDS NUMBER WITH MOMENTUM THICKNESS REYNOLDS NUMBER



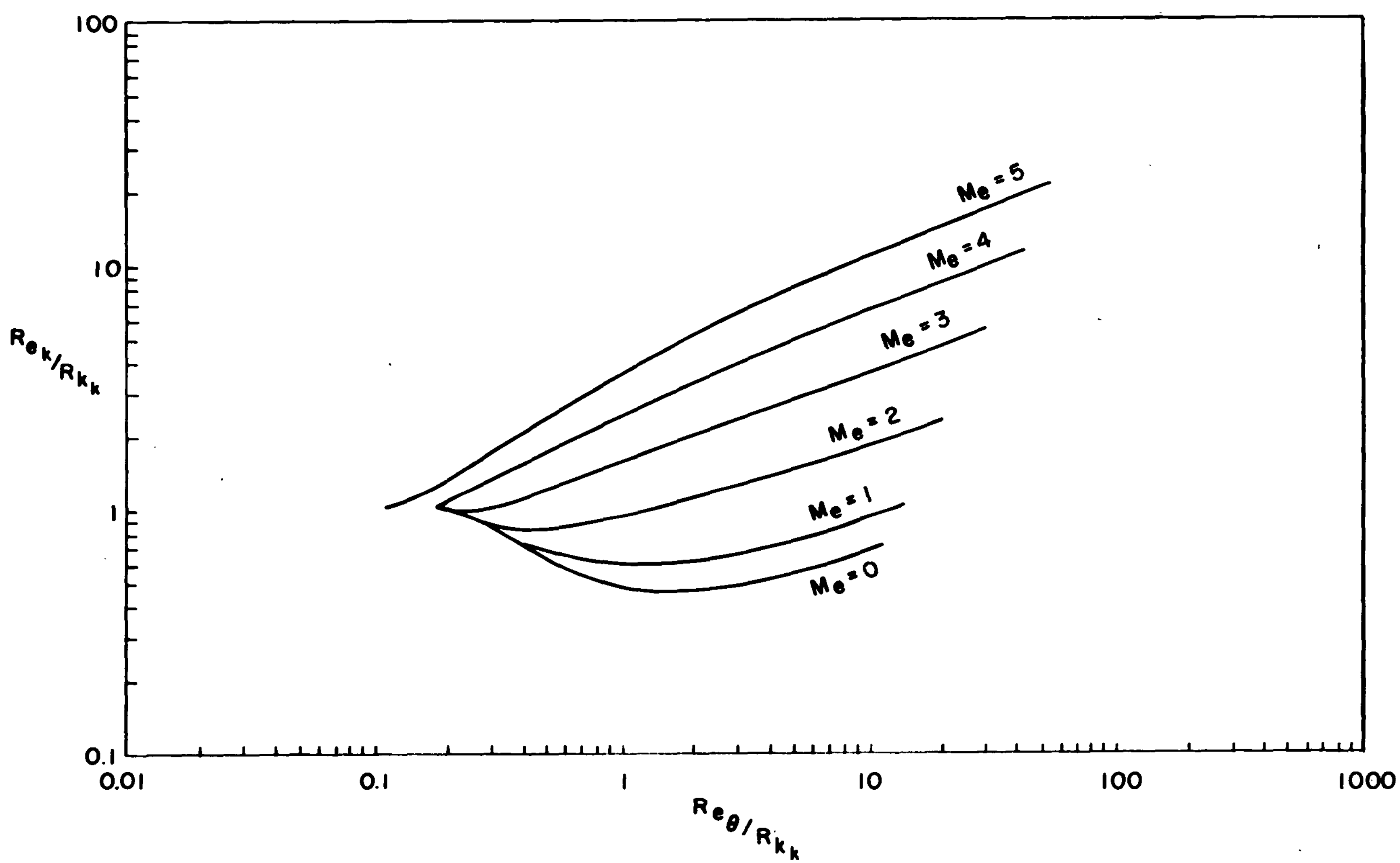

$S_w=-0.4$, $\beta=-0.2$

FIG. 1 (CONT.) VARIATION OF ROUGHNESS REYNOLDS NUMBER WITH MOMENTUM THICKNESS REYNOLDS NUMBER



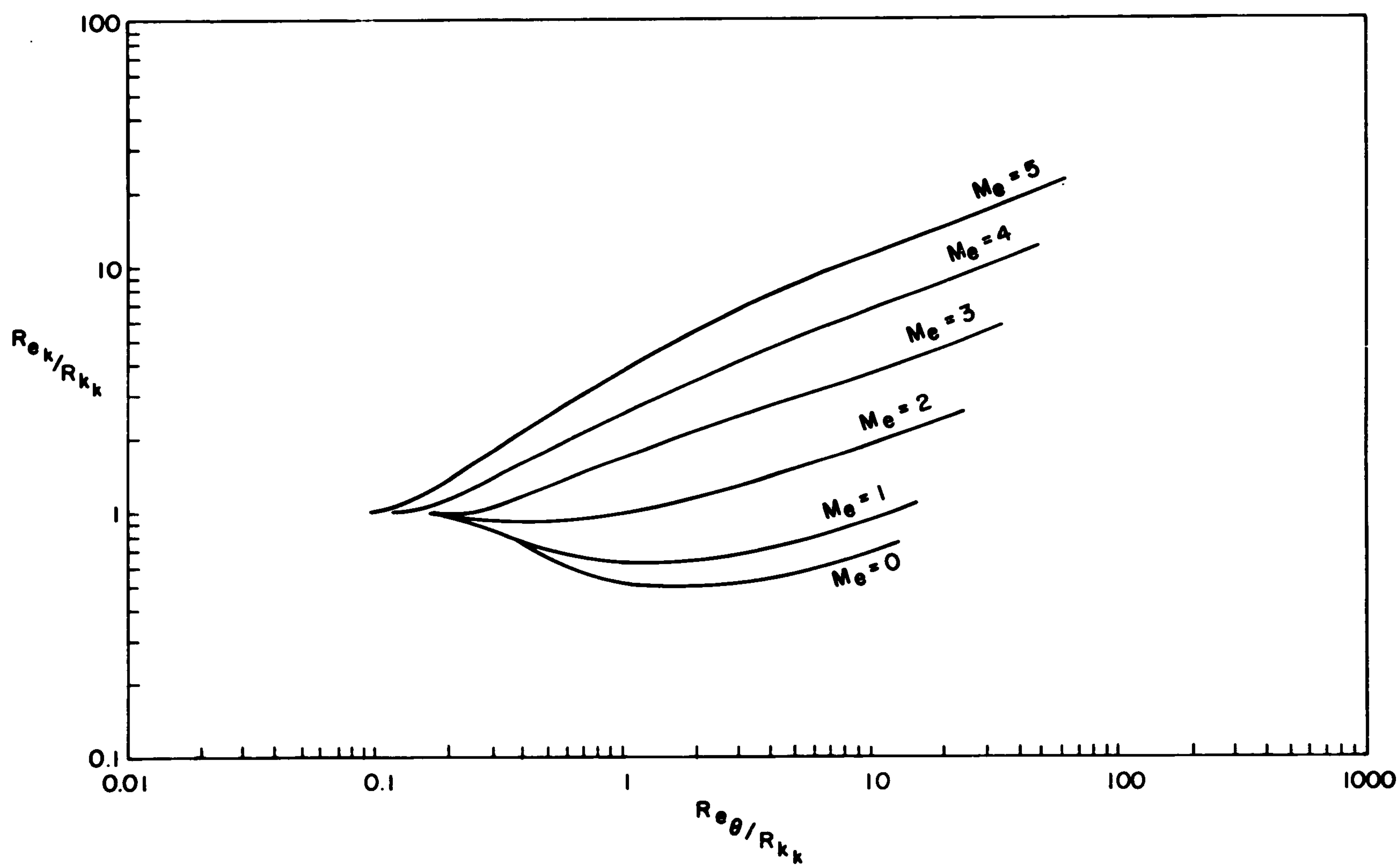

$S_w = -0.4$, $\beta = -0.24$

FIG. 1 (CONT.) VARIATION OF ROUGHNESS REYNOLDS NUMBER WITH MOMENTUM THICKNESS REYNOLDS NUMBER



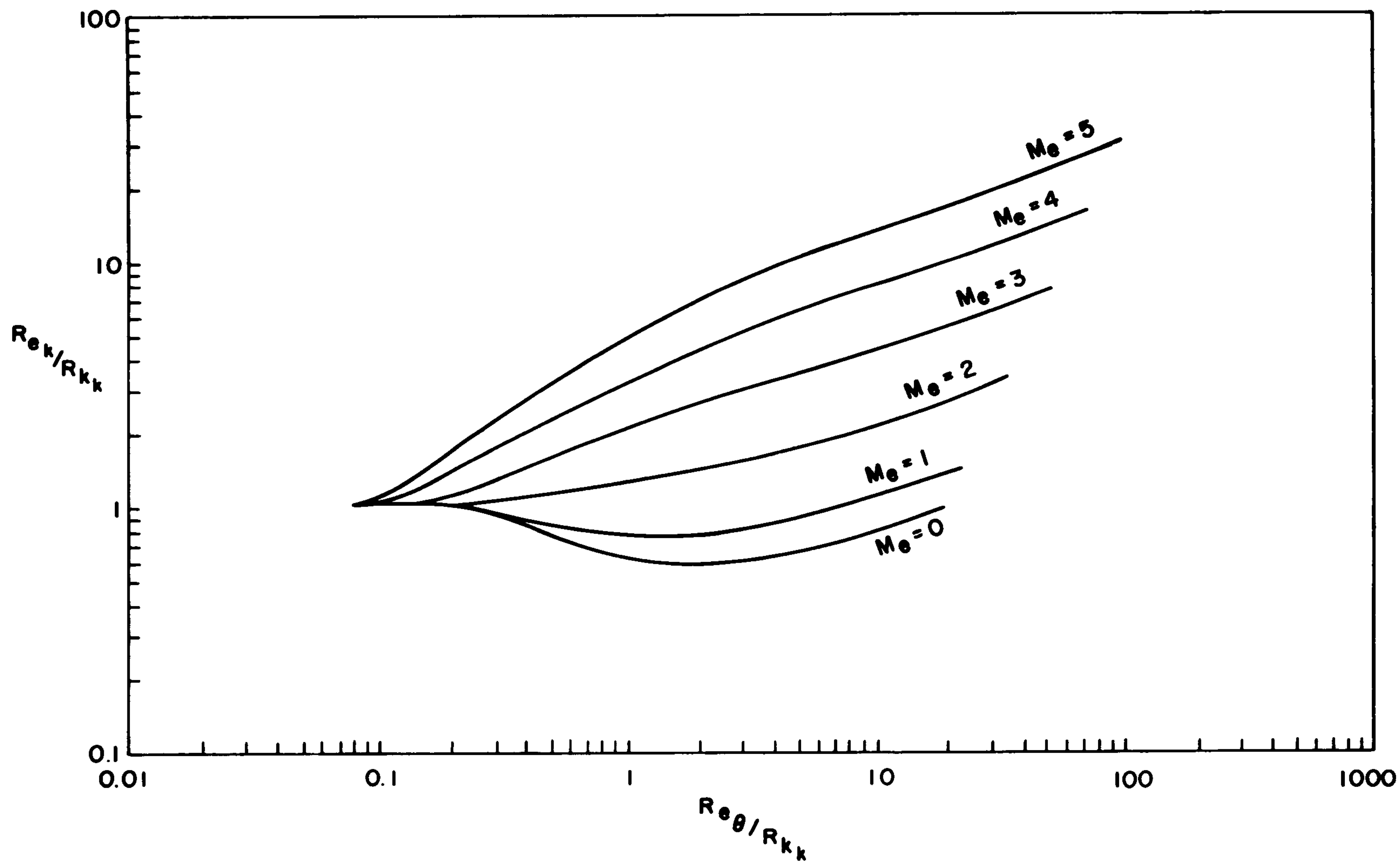

$S_w = -0.8$, $\beta = 2.0$

FIG. 1 (CONT.) VARIATION OF ROUGHNESS REYNOLDS NUMBER WITH MOMENTUM THICKNESS REYNOLDS NUMBER



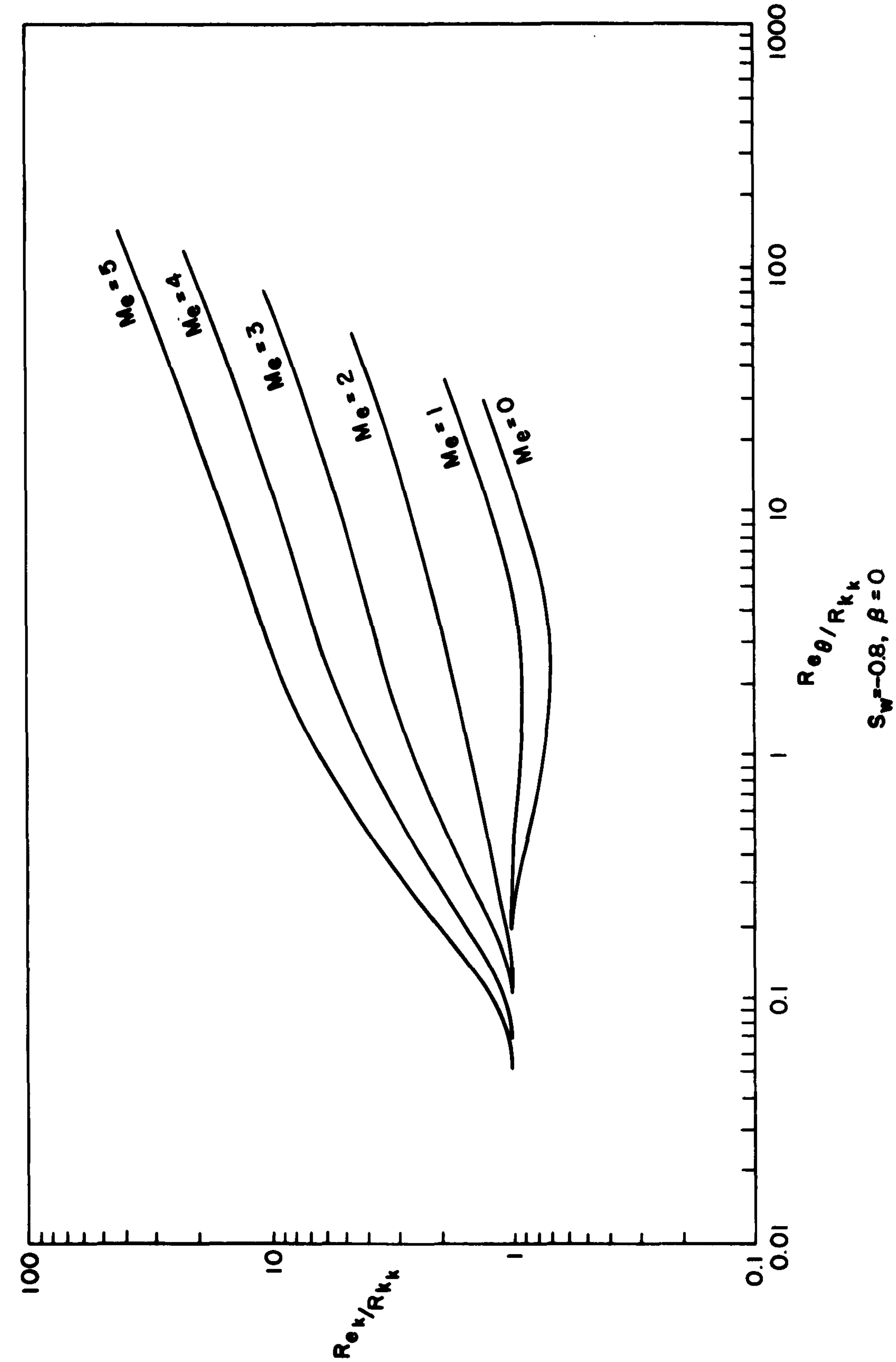

$S_w = -0.8, \ \beta = 1.5$

FIG. 1 (CONT.) VARIATION OF ROUGHNESS REYNOLDS NUMBER WITH MOMENTUM THICKNESS REYNOLDS NUMBER



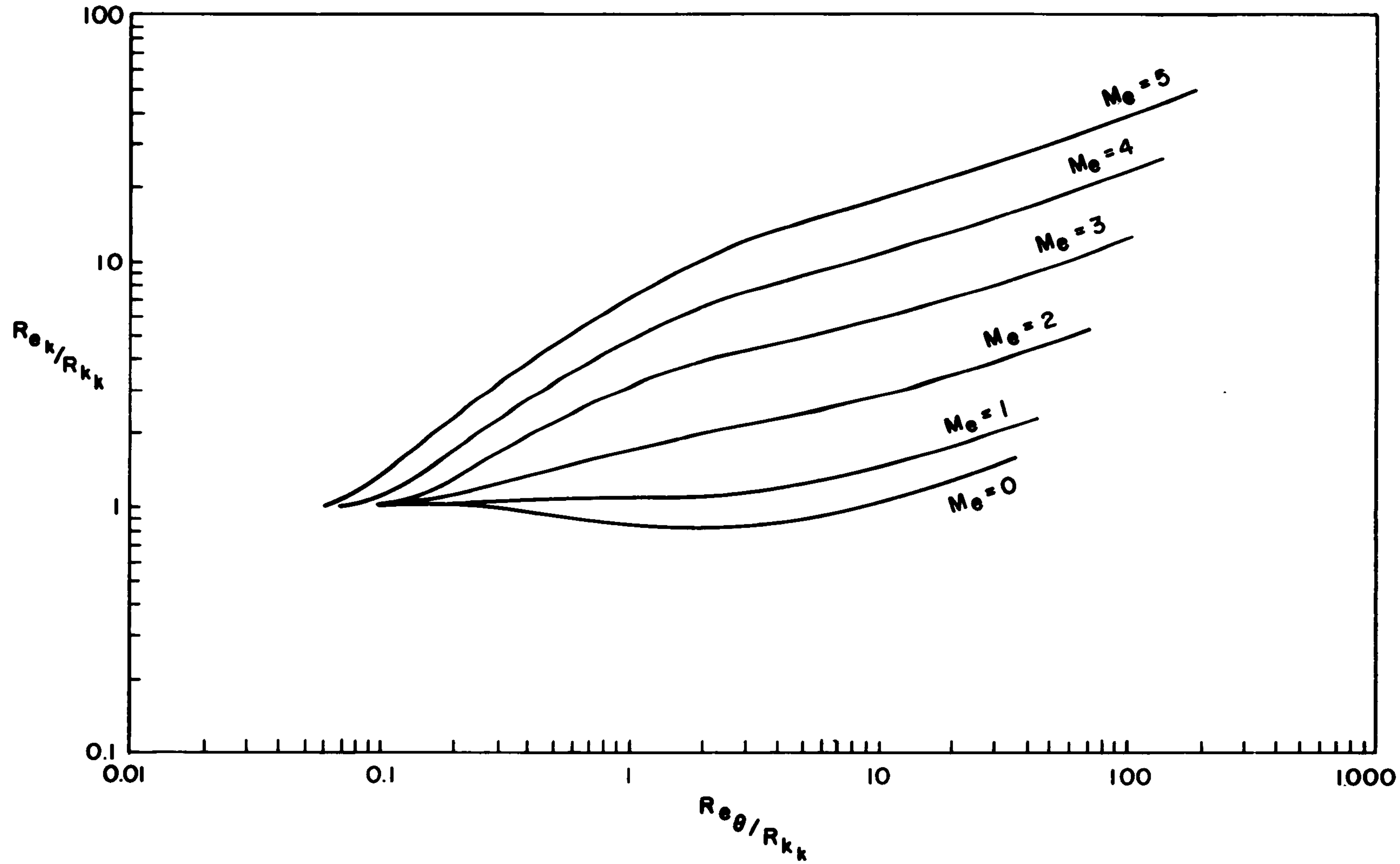

$S_w = -0.8$, $\beta = 0.5$

FIG. 1 (CONT.) VARIATION OF ROUGHNESS REYNOLDS NUMBER WITH MOMENTUM THICKNESS REYNOLDS NUMBER



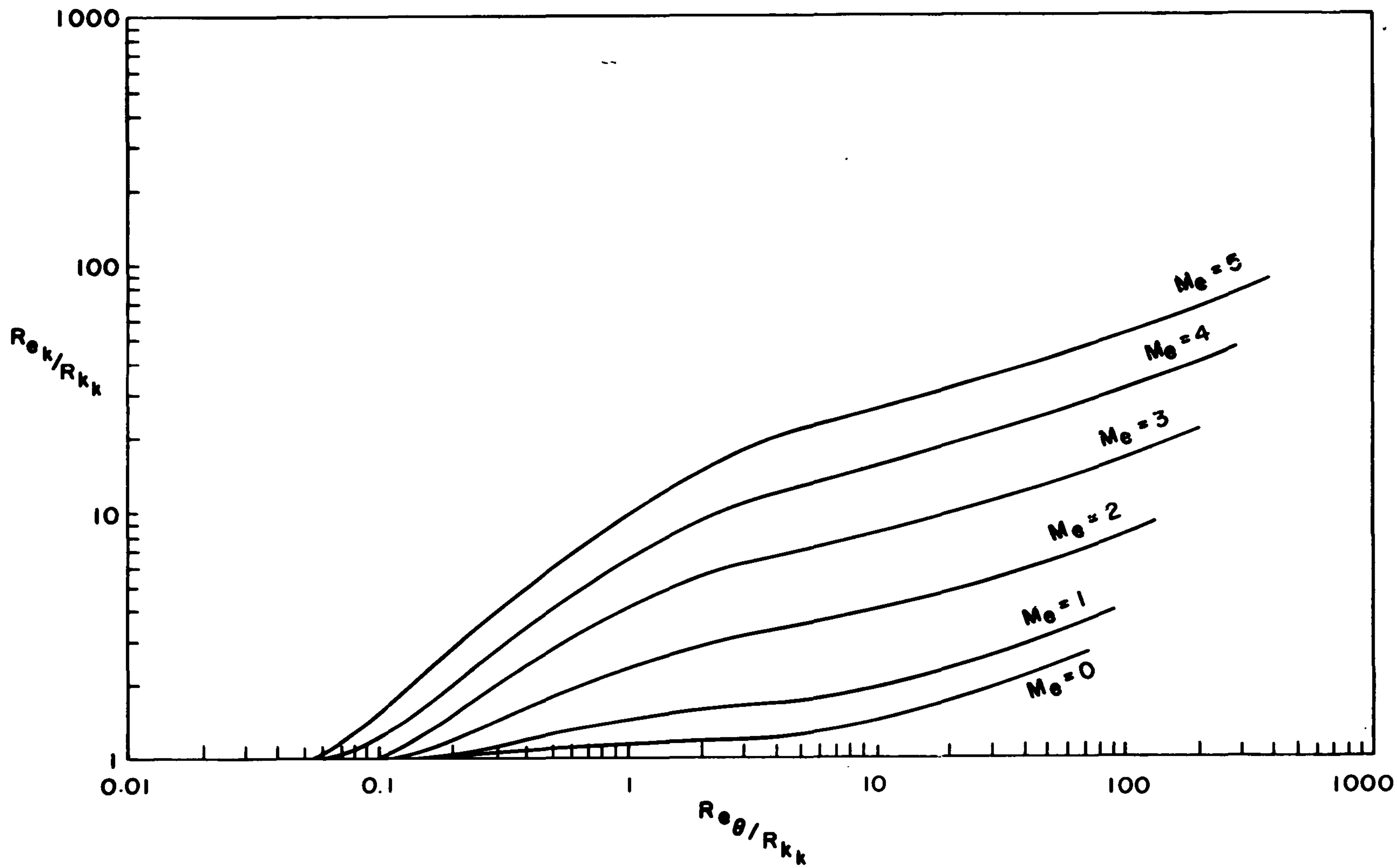

$S_w = -0.8,\ \beta = 0$

FIG. I (CONT.) VARIATION OF ROUGHNESS REYNOLDS NUMBER WITH MOMENTUM THICKNESS REYNOLDS NUMBER



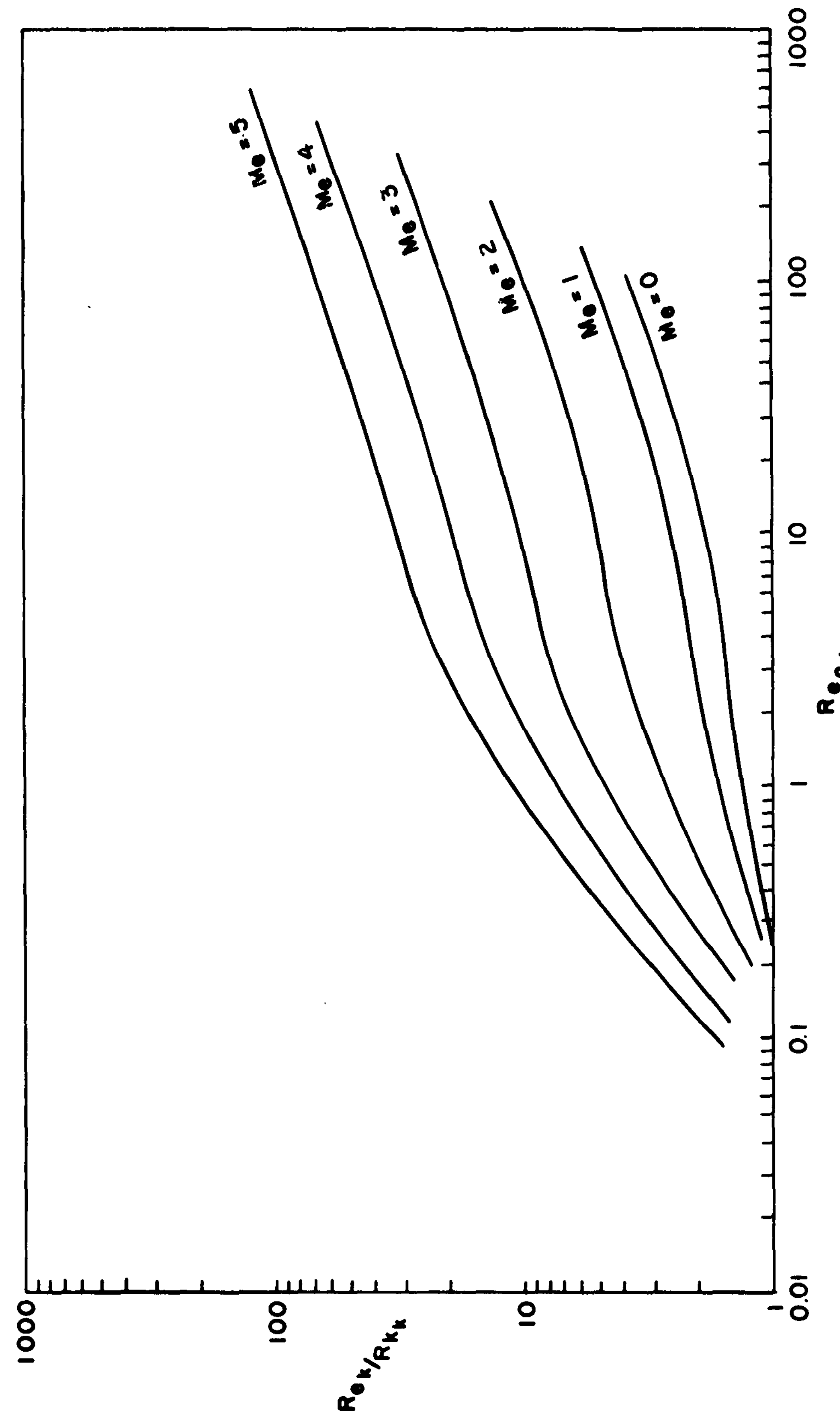

$S_w = -0.8$, $\beta = -0.14$

FIG. 1 (CONT.) VARIATION OF ROUGHNESS REYNOLDS NUMBER WITH MOMENTUM THICKNESS REYNOLDS NUMBER



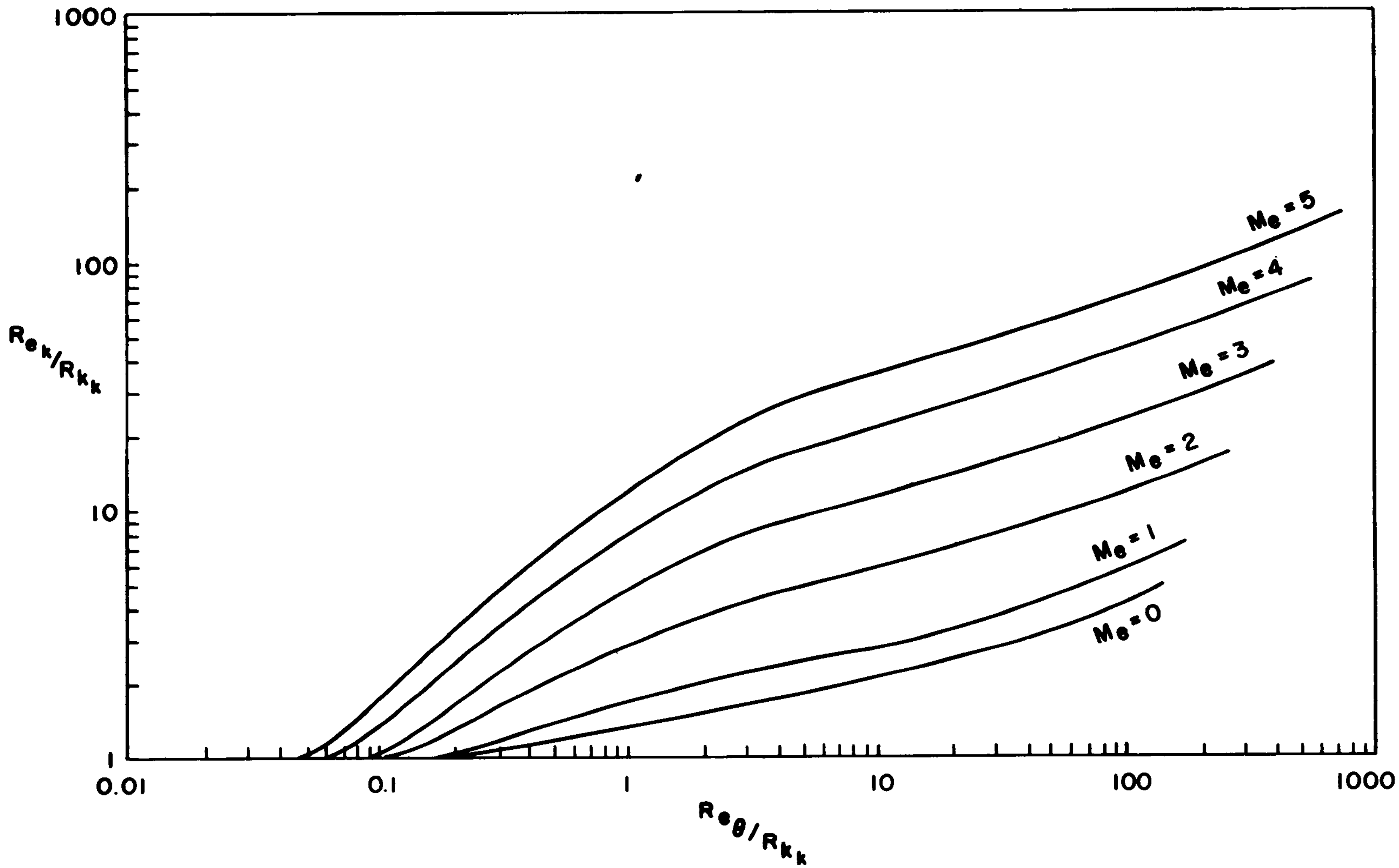

$S_w = -0.8$, $\beta = -0.3$

FIG. 1 (CONT.) VARIATION OF ROUGHNESS REYNOLDS NUMBER WITH MOMENTUM THICKNESS REYNOLDS NUMBER



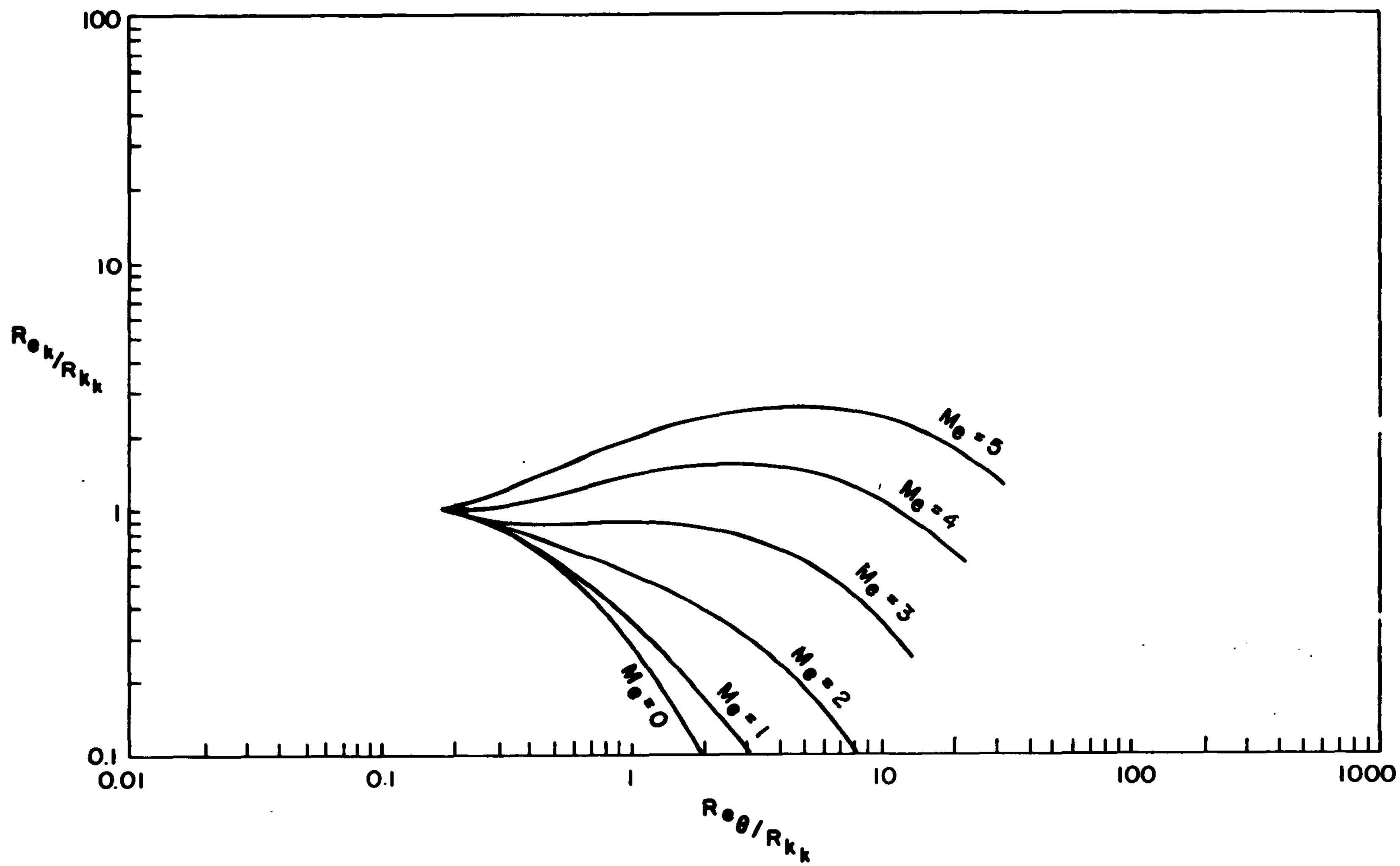

$S_w = -0.8,\ \beta = -0.325$

FIG. 1 (CONT.) VARIATION OF ROUGHNESS REYNOLDS NUMBER WITH MOMENTUM THICKNESS REYNOLDS NUMBER




$S_w = -0.8$, $\beta = -0.3285$

FIG. I (CONT.) VARIATION OF ROUGHNESS REYNOLDS NUMBER WITH MOMENTUM THICKNESS REYNOLDS NUMBER




$S_w = -1.0$, $\beta = 2.0$

FIG. 1 (CONT.) VARIATION OF ROUGHNESS REYNOLDS NUMBER WITH MOMENTUM THICKNESS REYNOLDS NUMBER

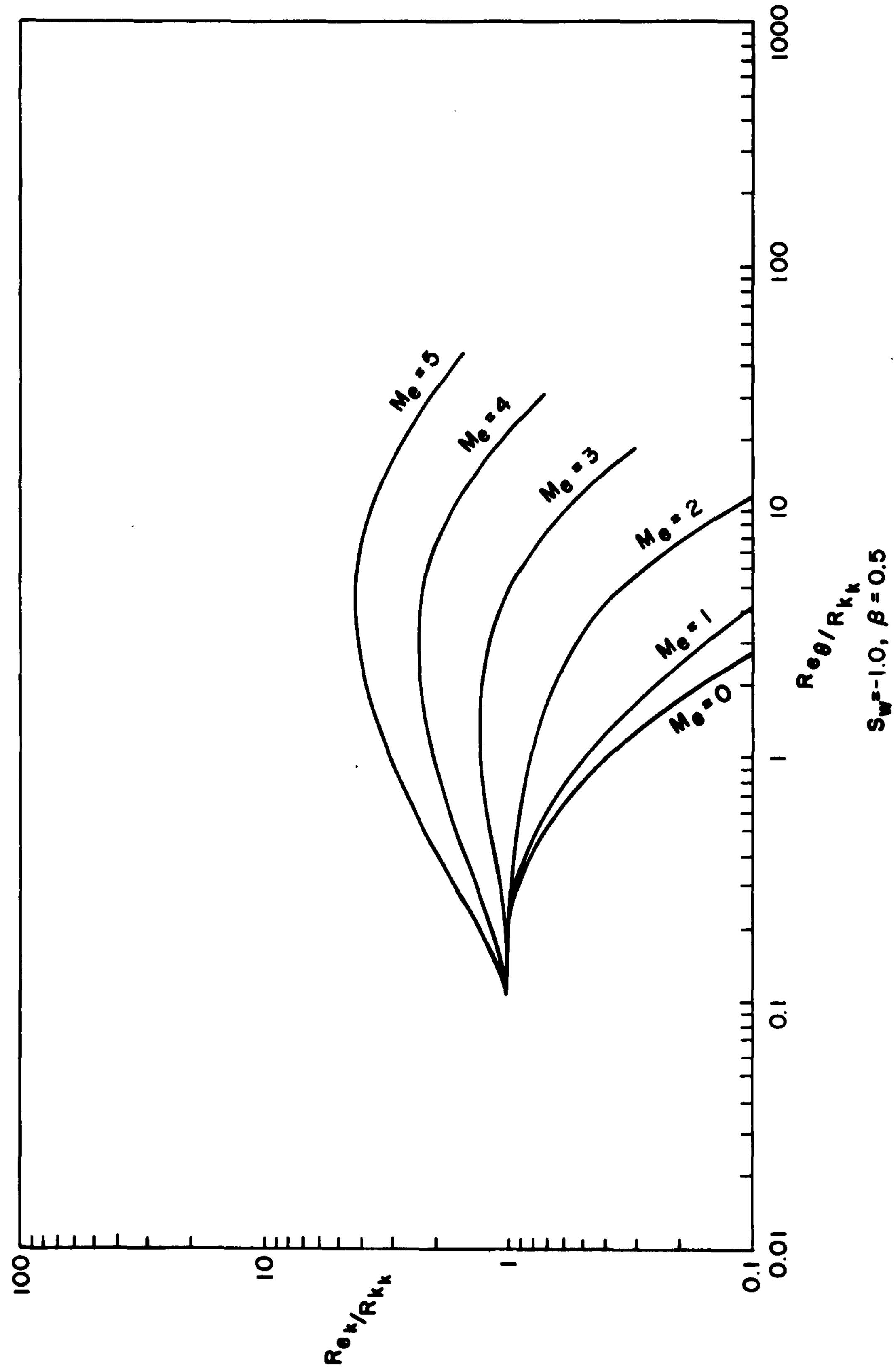
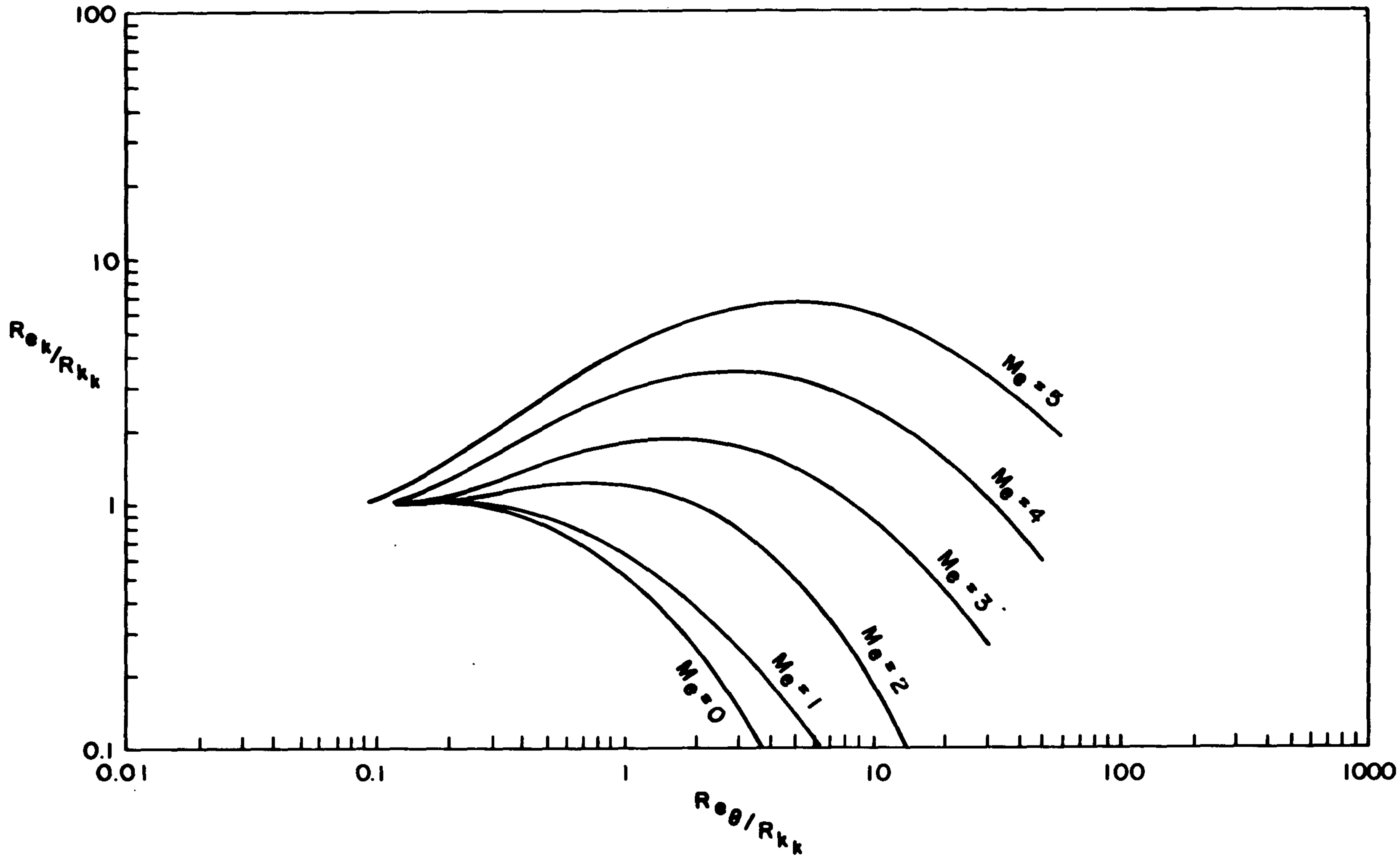



FIG. I (CONT.) VARIATION OF ROUGHNESS REYNOLDS NUMBER WITH MOMENTUM THICKNESS REYNOLDS NUMBER




$S_w = -1.0$, $\beta = 0$

FIG. I (CONT.) VARIATION OF ROUGHNESS REYNOLDS NUMBER WITH MOMENTUM THICKNESS REYNOLDS NUMBER

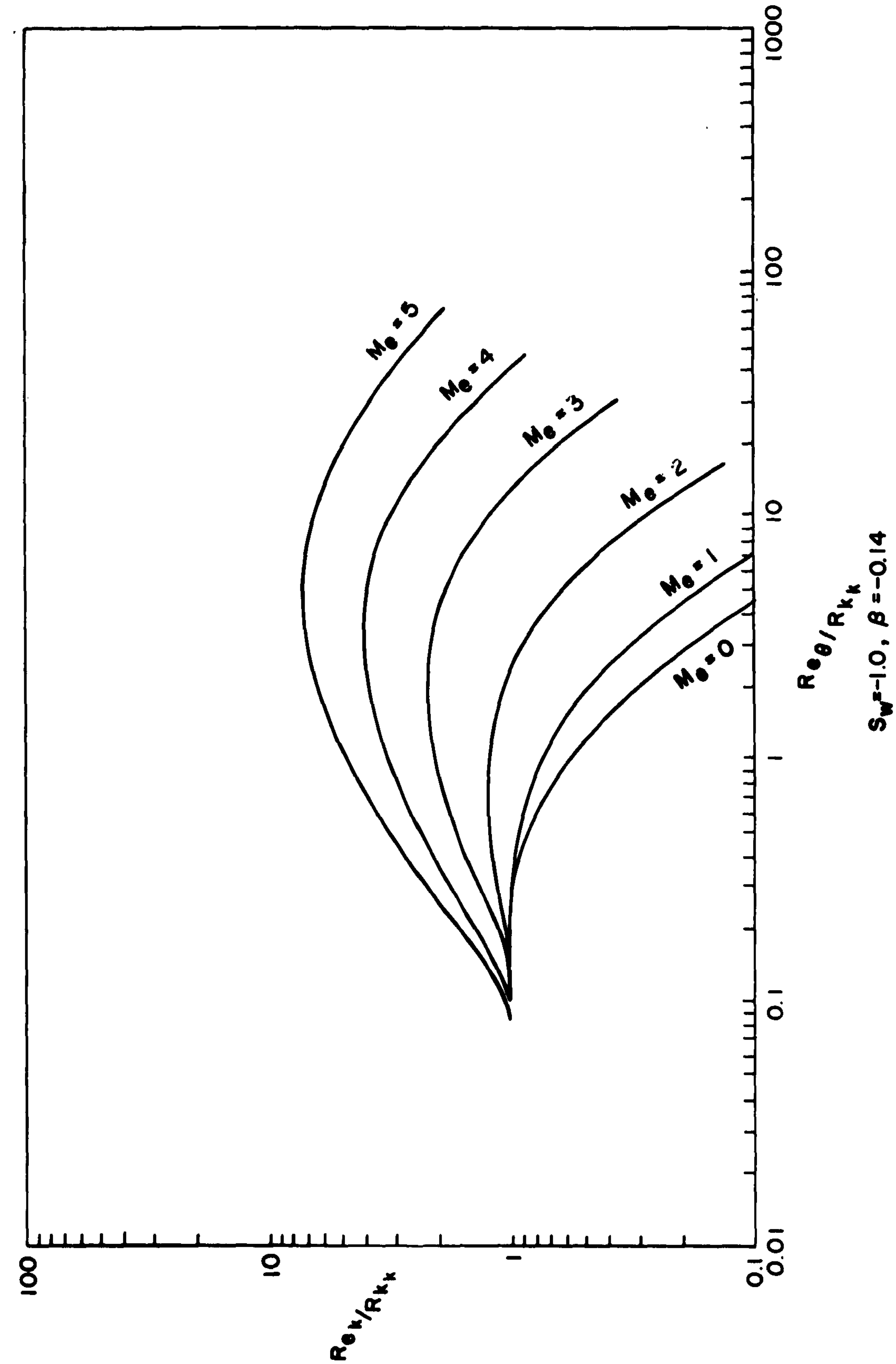
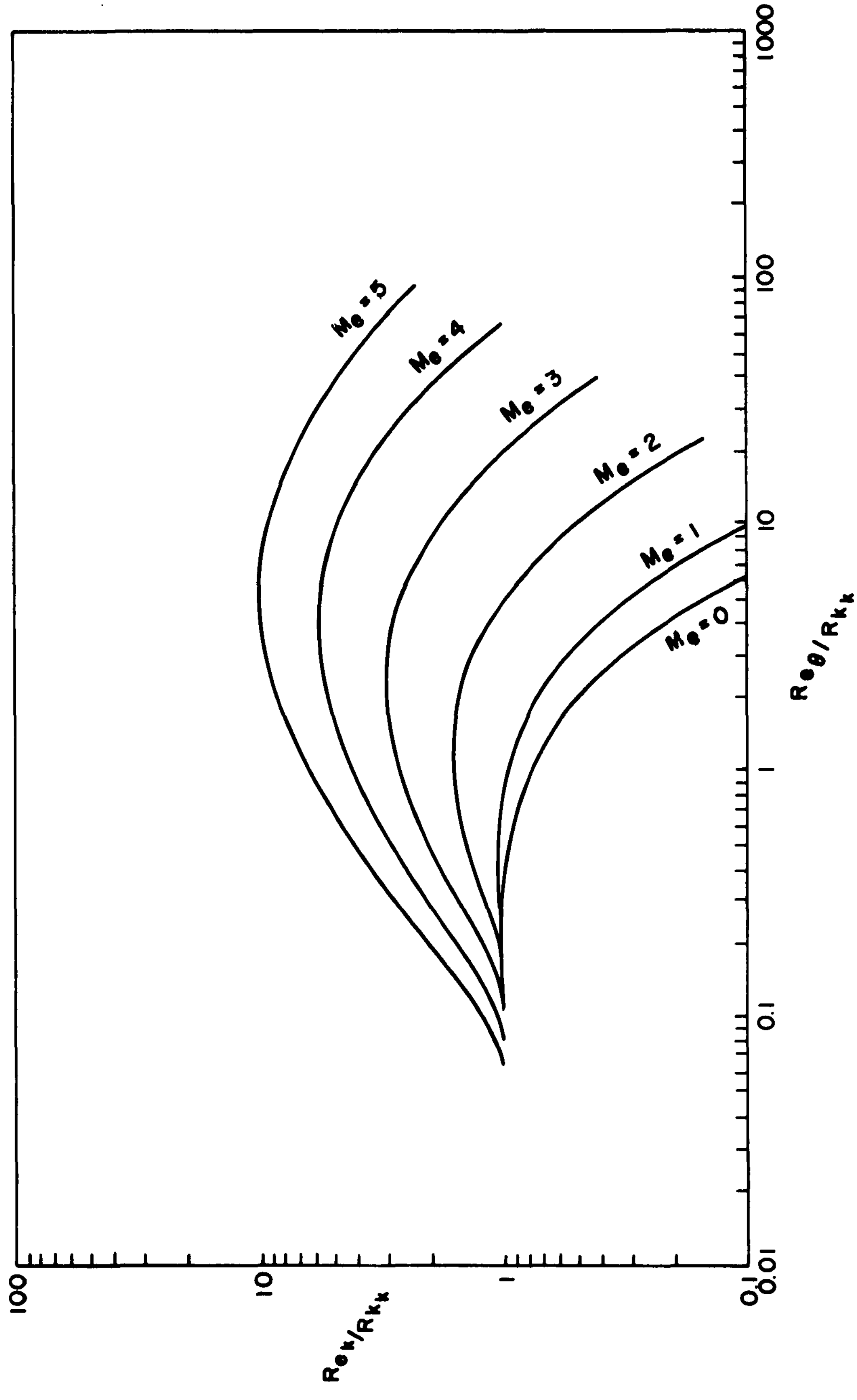
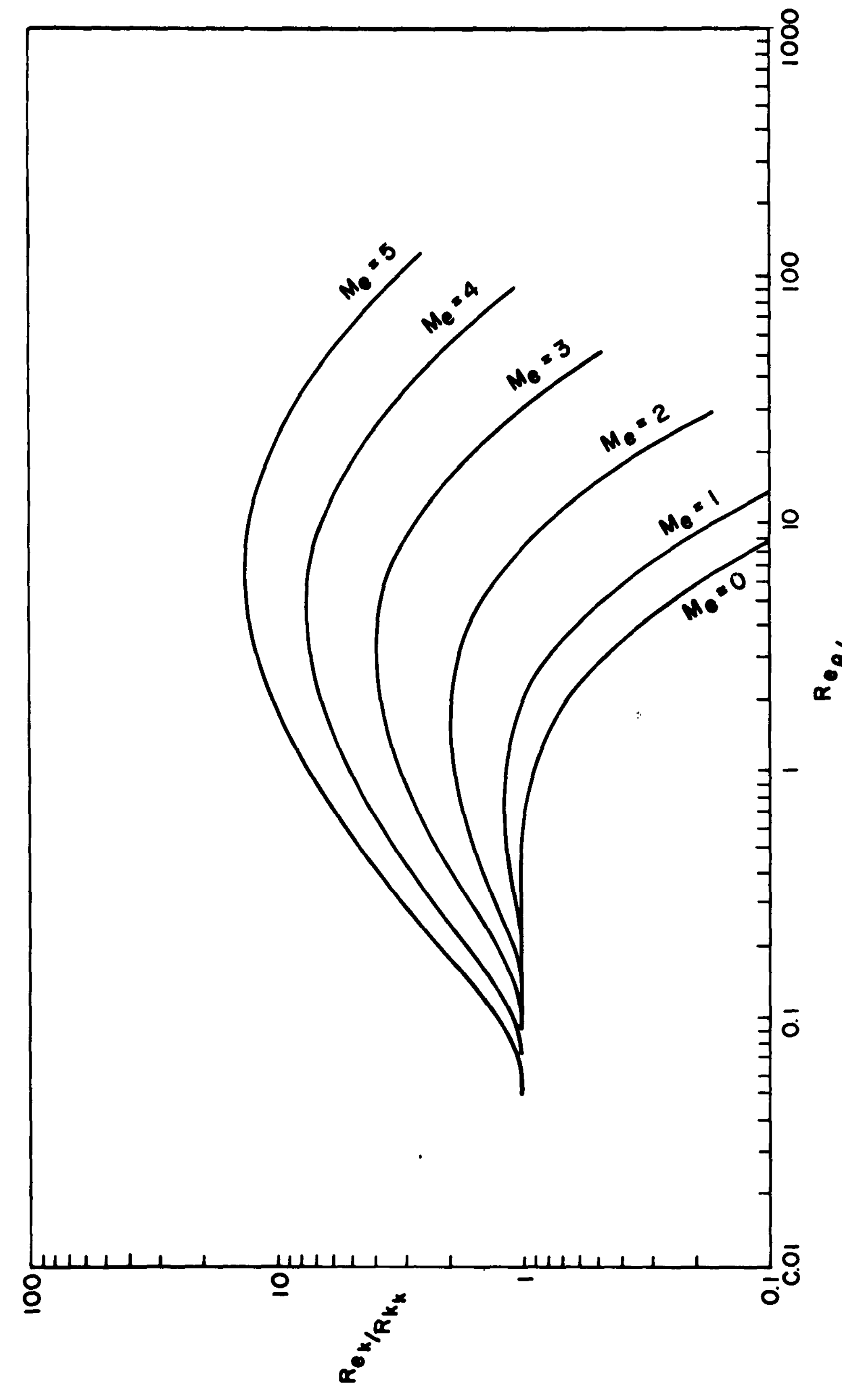



FIG. 1 (CONT.) VARIATION OF ROUGHNESS REYNOLDS NUMBER WITH MOMENTUM THICKNESS REYNOLDS NUMBER



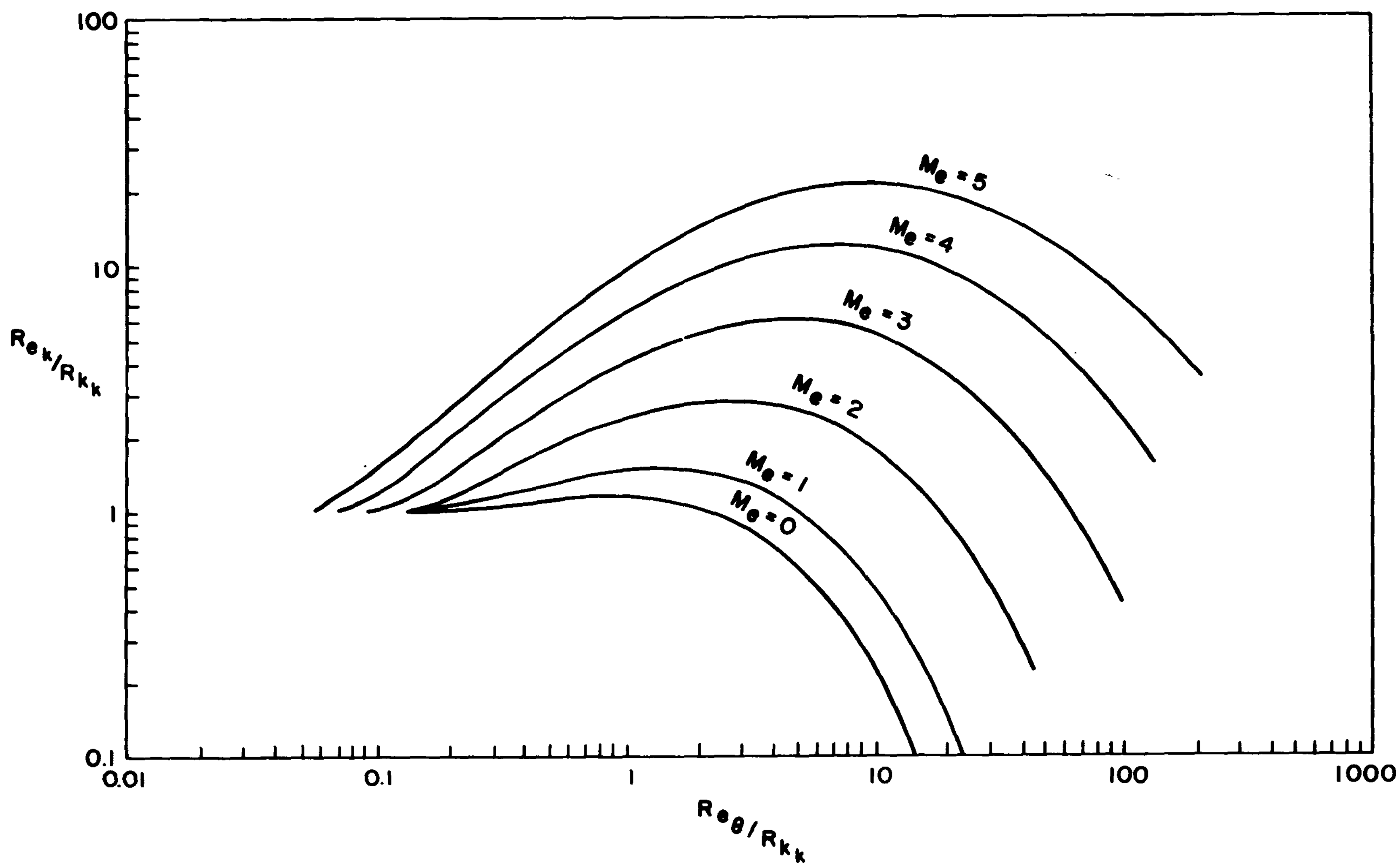

$S_w = -1.0, \ \beta = -0.3$

FIG. I (CONT.) VARIATION OF ROUGHNESS REYNOLDS NUMBER WITH MOMENTUM THICKNESS REYNOLDS NUMBER




$S_w = -1.0$, $\beta = -0.36$

FIG. 1 (CONT.) VARIATION OF ROUGHNESS REYNOLDS NUMBER WITH MOMENTUM THICKNESS REYNOLDS NUMBER




$S_w = -1.0,\ \beta = -0.3884$

FIG. 1 (CONT.) VARIATION OF ROUGHNESS REYNOLDS NUMBER WITH MOMENTUM THICKNESS REYNOLDS NUMBER

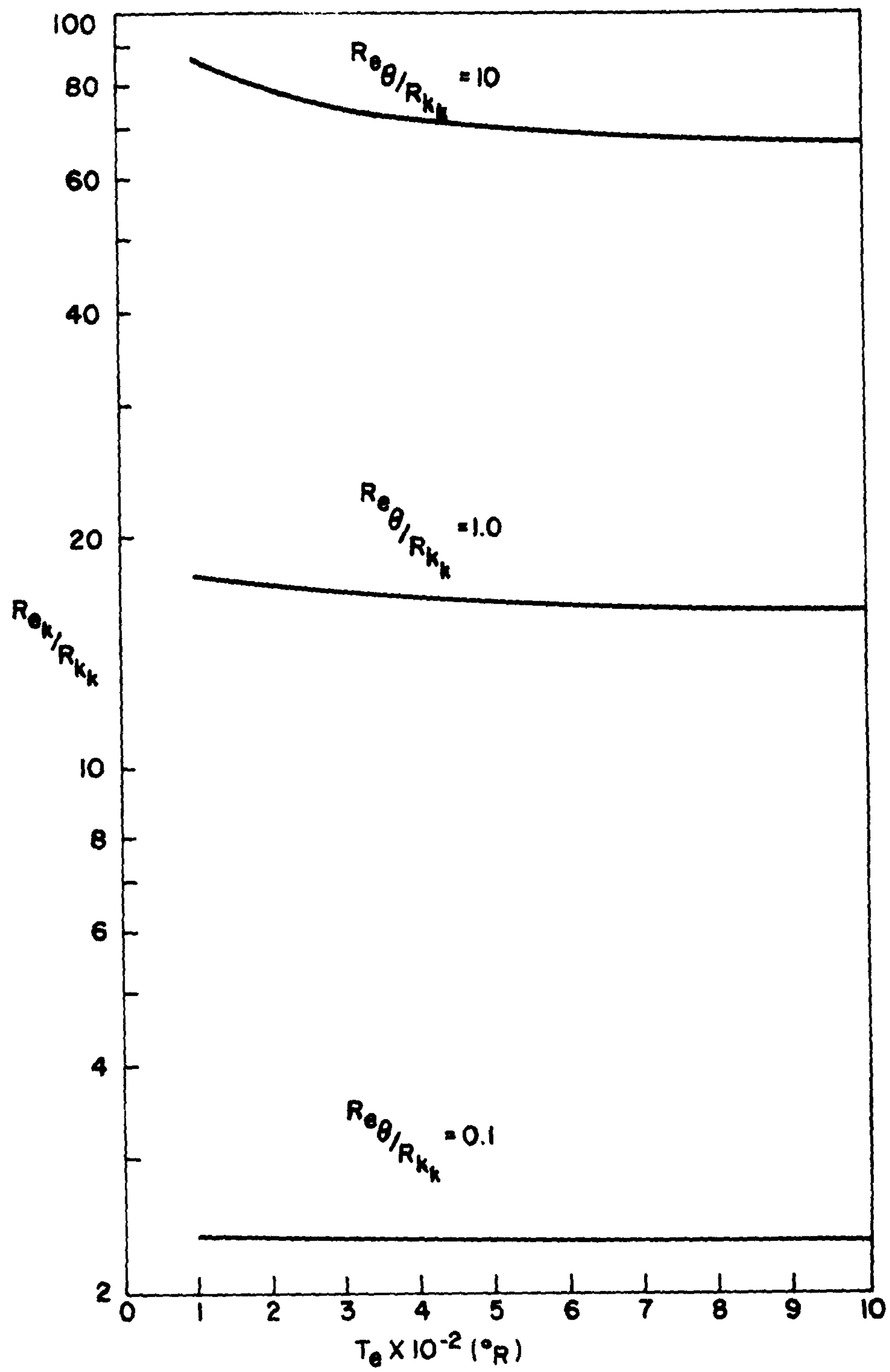



FIG. 2 VARIATION OF ROUGHNESS REYNOLDS NUMBER WITH LOCAL TEMPERATURE

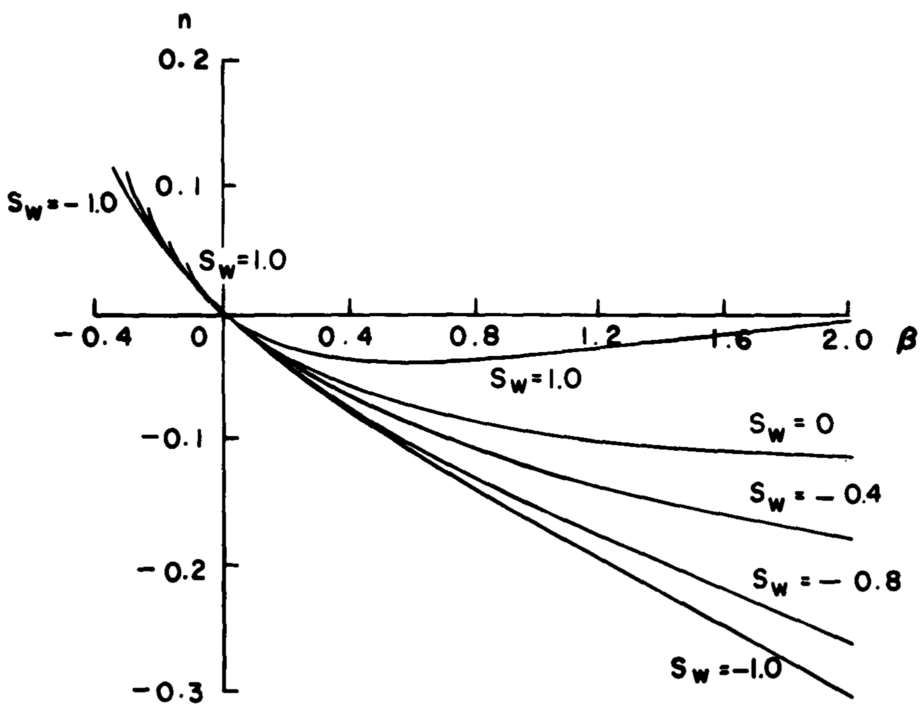



FIG. 3 VARIATION OF PRESSURE GRADIENT PARAMETER $\beta$ WITH THE CORRELATION NUMBER $n$

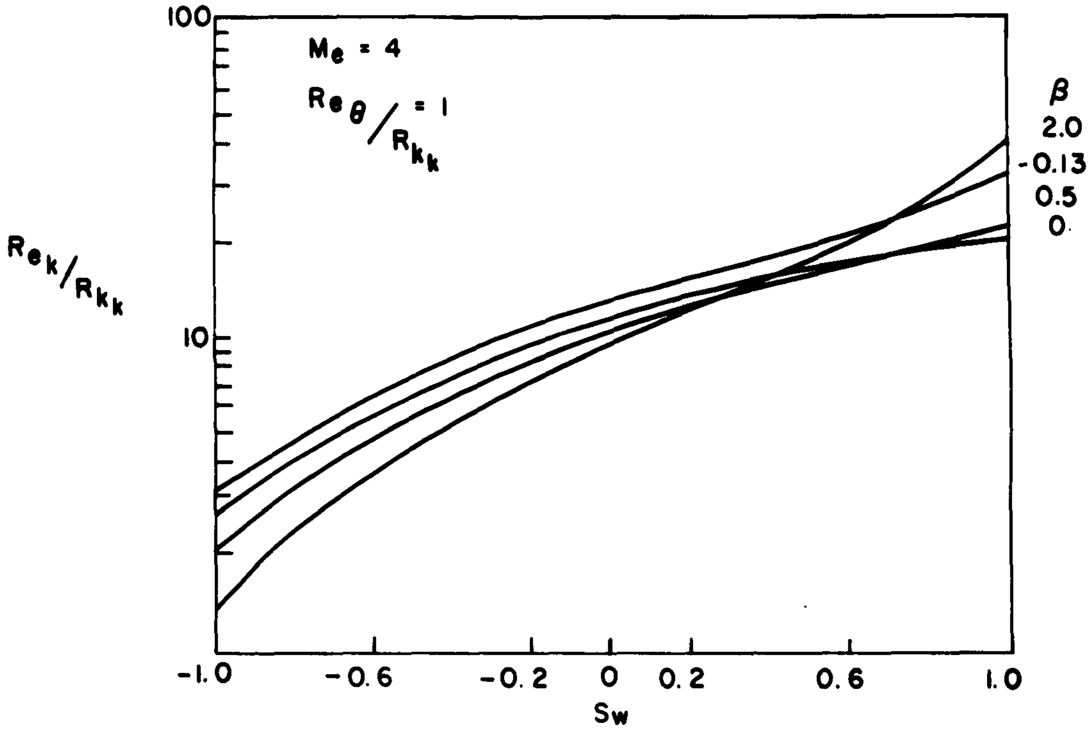



FIG. 4 VARIATION OF ROUGHNESS REYNOLDS NUMBER WITH $S_w$

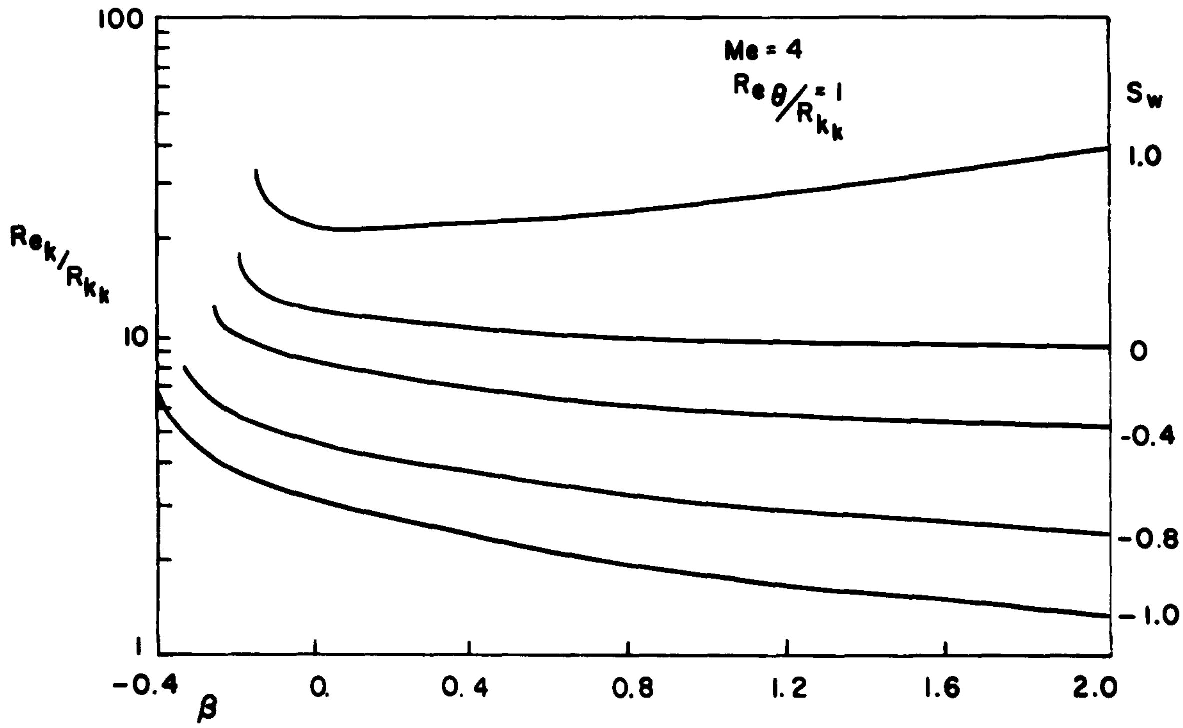



FIG. 5 VARIATION OF ROUGHNESS REYNOLDS NUMBER WITH $\beta$

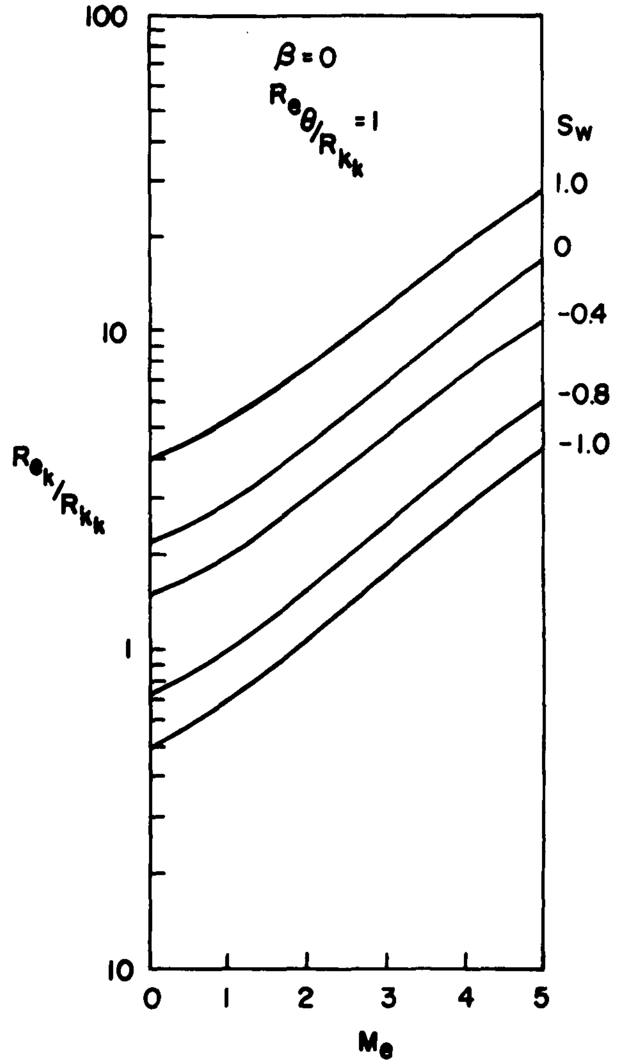



FIG. 6 VARIATION OF ROUGHNESS REYNOLDS NUMBER WITH $M_e$

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| Turbulent | TUBU | | | | |
| Flow | FLOW | | | | |
| Roughness | ROUG | | | | |
| Transition | TRNI | | | | |
| Reynolds number | REYN | | | | |
| Flow (Properties) | FLOWP | | | | |
| Experimental | EXPE | | | | |
| Parameters | PARA | | | | |
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1. Boundary layer - Transition
2. Boundary layer, Turbulent
I. Title
II. Lyons, W. Carson
III. Levensteins, Zigurds J., jt. author
IV. Series
V. Project

Naval Ordnance Laboratory, White Oak, Md.
(NOL technical report 61-87)
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